Math 202 Major 2

Prob. 1: (13 Points)
Solve the initial value problem on the interval $(-\infty, 0)$

$$4x^2y'' + y = 0;\ y(-1) = 2,\ y'(-1) = 4$$

Prob. 2: (13 Points)
Solve the differential equation

$$2y'' - 4y' + 2y = e^x \ln x,\ x > 0.$$

Prob. 3: (16 Points)
(a) Find three linearly independent functions that are annihilated by the differential operator

$$D^3 - 8 \ where \ D = \frac{d}{dx}.$$

(b) Use the annihilator approach to solve the differential equation

$$y'' - 9y = 2e^{5x} - 8\cos(2x).$$

(Do not evaluate the constants!)

Prob. 4: (14 Points)
Solve the initial value problem

$$y''' - 5y'' + 100y' - 500y = 0;\ y(0) = 0,\ y'(0) = 10,\ y''(0) = 250$$

given that $y_1(x) = e^{5x}$ is a solution of the differential equation.

Prob. 5: (11 Points)
Let $y_1 = x^{-1/2}\sin x$ be a solution of $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$. Use the reduction of order method to find a second solution.

Prob. 6: (11 Points)

Consider the differential equation

$$y'' + y = \sec x + e^x$$

(a) Check that $x \sin x + (\cos x) \ln(\cos x)$ is a particular solution of

$$y'' + y = \sec x.$$

(b) Find the general solution of $y'' + y = \sec x + e^x$.

Prob. 7: (11 Points)

Show that x, $x \ln x$ and x^2 form a fundamental set of solutions (are solutions and are linearly independent) of the differential equation

$$x^3 y''' - x^2 y'' + 2xy' - 2y = 0, \ x > 0.$$

Prob. 8: (11 Points)

Let $y = C_1 \cos \omega x + C_2 \sin \omega x$, $\omega \neq 1$, be a 2-parameter family of solutions of the differential equation $y'' + \omega^2 y = 0$. Determine whether a member of the family can be found that satisfies the boundary conditions $y(0) = 1$ and $y'(\frac{\pi}{2\omega}) = -1$.