MATH 202.18 (Term 102)

Quiz 2 (Chap. 2.3-3.1) Duration: 20mn

Name: **ID number:**

1.)(5pts)Solve the exact DE: $(\sin^2(x+y)+\frac{x^2y^3}{3})dx+(-\frac{1}{2}\cos 2(x+y)+\frac{x^3y^2}{3})dy=0.$

2.)(5pts)Solve by substitution the DE: $\frac{dy}{dx}=\frac{x-y+5}{x-y}.$

Solution

1.) $M=\sin^2(x+y)+x^2y^3/3$

$\Rightarrow M_y = 2\cos(x+y)\sin(x+y)+x^2y^2$

$= \sin 2(x+y)+x^2y^2$

$N=-\frac{1}{2}\cos 2(x+y)+x^3y^2/3$

$\Rightarrow N_x = \sin 2(x+y)+x^2y^2$

$M_y=N_x \Rightarrow$ DE exact.

$\frac{\partial f}{\partial x}=\sin^2(x+y)+\frac{x^2y^3}{3}$, $\frac{\partial f}{\partial y}=-\frac{1}{2}\cos 2(x+y)+\frac{x^3y^2}{3}$

$f(x,y)=\frac{-\sin 2(x+y)}{4}+\frac{x^3y^3}{9}+h(x)$

$f_x = -\frac{\cos 2(x+y)}{2}+\frac{x^2y^3}{3}+h'(x)=$

$\sin^2(x+y)+\frac{x^2y^3}{3}$

Noting that $\sin^2(x+y)=\frac{1-\cos 2(x+y)}{2}$,

we find $h'(x)=\frac{1}{2}$

$h(x)=\frac{x}{2}+C$

$\boxed{\frac{-\sin 2(x+y)}{4}+\frac{x^2y^3}{3}+\frac{x}{2}=C}$

2.) $\frac{dy}{dx}=\frac{x-y+5}{x-y}$

$u=x-y \Rightarrow \frac{du}{dx}=1-\frac{dy}{dx}$

We substitute into the DE, and we find

$1-\frac{du}{dx}=\frac{u+5}{u}$

$\frac{du}{dx}=1-\frac{u+5}{u}=-\frac{5}{u}$

$\int u\,du=-\int 5\,dx$

$u^2=-5x+C$

$\boxed{(x-y)^2=-5x+C}$