MATH 202.15 (sem. 112)

Quiz 1 (Sects. 1.1-2.2) Duration: 20mn

Name: ID number:

1.)(6pts)Explain whether the following IVP have unique solutions?

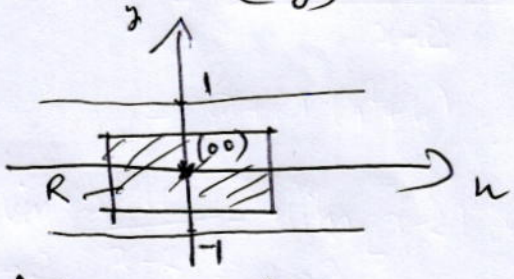
a.) $\begin{cases} y' = \frac{x-1}{1-y^2} \\ y(0) = 0, \end{cases}$ b.) $\begin{cases} y' = \sqrt{9-x^2-y^2} \\ y(3) = 0. \end{cases}$

2.)(4pts)Solve the separable DE: $(x^2-4)\cos^2 y dy = dx.$

1.) a) $\frac{dy}{dx} = f(x,y)$, where

$f(x,y) = \frac{x-1}{1-y^2}$, $y \neq \pm 1$

$\frac{\partial f}{\partial y}(x,y) = \frac{2y(x-1)}{(1-y^2)^2}$, $y \neq \pm 1$



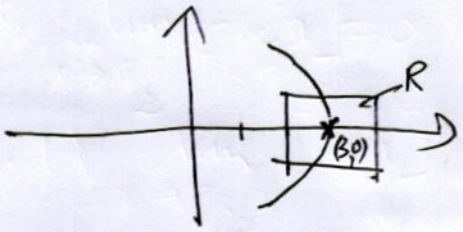
$f(x,y)$ and $\frac{\partial f}{\partial y}(x,y)$ are continuous on R.

Thus, the IVP has a unique solution y on $(-\infty, \infty)$.

b.) $\frac{dy}{dx} = f(x,y)$, where

$f(x,y) = \sqrt{9-x^2-y^2}$, $x^2+y^2 \leq 9$

$\frac{\partial f}{\partial y}(x,y) = \frac{-2y}{2\sqrt{9-x^2-y^2}}$, $x^2+y^2 < 9$



$\frac{\partial f}{\partial y}(x,y)$ is not continuous on R.

So, we can't say whether the IVP has unique solution or not.

2.) $(x^2-4)\cos^2 y \, dy = dx$

$\int \cos^2 y \, dy = \int \frac{dx}{x^2-4}$

$\int \frac{1+\cos 2y}{2} dy = \frac{1}{4}\int \left(\frac{1}{x-2} - \frac{1}{x+2}\right) dx$

$\frac{y}{2} + \frac{\sin 2y}{4} = \frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + C$

$= \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$

$2y + \sin 2y = \ln\left|\frac{x-2}{x+2}\right| + C$

$$\boxed{\frac{x-2}{x+2} = Ce^{2y+\sin 2y}}, \quad x \neq -2$$