April 28,2012

QUIZ♯3 Math202, sec 11

Net Time Allowed: 20 minutes

Name: **ID ♯ :** **Serial:**

Exercise1:

Find the recurrence relation determining the coefficients of the power series solution:

$$y'' - xy' + 3y = 0 \tag{1}$$

Solution:

• Clearly O is an ordinary point.

• Let $y = \sum_{n=0}^{\infty} C_n x^n$, so $y' = \sum_{n=1}^{\infty} nC_n x^{n-1}$, $y'' = \sum_{n=2}^{\infty} n(n-1)C_n x^{n-2}$

Thus $y'' - xy' + 3y = \sum_{n=2}^{\infty} n(n-1)C_n x^{n-2} - \sum_{n=1}^{\infty} nC_n x^n + \sum_{n=0}^{\infty} 3C_n x^n = 0$

so $\sum_{k=0}^{\infty}(k+2)(k+1)C_{k+2}x^k - \sum_{k=1}^{\infty} kC_k x^k + 3c_0 + 3\sum_{k=1}^{\infty} C_k x^k = 0$

$$2c_2 + 3c_0 + \sum_{k=1}^{\infty}[(k+1)(k+2)c_{k+2} - kc_k + 3c_k]x^k = 0$$

$$\Rightarrow \begin{cases} 3c_0 + 2c_2 = 0 \\ (k+1)(k+2)C_{k+2} + (3-k)C_k = 0, \ k = 1, 2, \ldots \end{cases}$$

$$\text{Hence} \Rightarrow \begin{cases} c_2 = -\dfrac{3}{2}c_0 \\ c_{k+2} = \dfrac{(k-3)}{(k+1)(k+2)}c_k, \ k = 1, 2, 3, \ldots \end{cases}$$

Exercise2:

Consider the differential equation: $$4xy'' + \frac{1}{2}y' + y = 0, \tag{1}$$

1. Show that x=0 is a regular singular point.
2. Find the indicial equation and indicial roots of the differential equation (1).