Major #1 Math 202

February 20

Prob. 1:
(a) Verify that $y = \frac{C}{e^{-x}+C}$ is a family of solutions of the DE $\frac{dy}{dx} = y(1-y)$
(b) What is the order of

$$2xy \left(\frac{dy}{dx}\right)^{10} = 3x^{20} \left(\frac{d^3y}{dx^3}\right)^5 + x + y^{20} - 6$$

Prob. 2:
(a) Verify that $y = -\frac{1}{x+C}$ is a one-parameter family of solutions of the DE $y' = y^2$
(b) Find a solution from the family in (a) that satisfies $y(0) = -1$.
(c) Determine the largest interval of definition of the solution

Prob. 3:
Solve the initial value problem

$$e^{-2x}\frac{dy}{dx} = (y - y^2)x, \ y(0) = 1/2.$$

Prob. 4:
Solve the ODE

$$e^x \frac{dy}{dx} - 2xe^x y = 2x + 1$$

Prob. 5:
Solve the DE

$$[2x + y\cos(xy)]\, dx + x\cos(xy)\, dy = 0$$

Prob. 6:

Find an integrating factor for

$$(xy \cos y - 2x \sin y)\, dy + 2y \sin y\, dx = 0$$

Prob. 7:
Solve the initial value problem

$$x\frac{dy}{dx} + (x-2)y = \frac{y^2 \sin x}{x}, \ y(\pi) = 1$$

Prob. 8:
The population of a town grows at a rate proportional to the population present at time t. The initial population of 1000 increases by 30% in 30 years. What will be the population in 40 years?