King Fahd University of Petroleum and Minerals
Department of Math & Stat
Math 201 Section # 6 (112)
Quiz 1(a)

Time:20 minutes

Marks:________/8

Name: "Solution" ID#:________ Serial#:________

1. Let $r = 5\cos 3\theta$.

(a) Check symmetry of this polar curve about $x-axis, y-axis$ and the origin:

$r = 5\cos 3(-\theta) = 5\cos 3\theta \Rightarrow$ curve is symmetric about x-axis

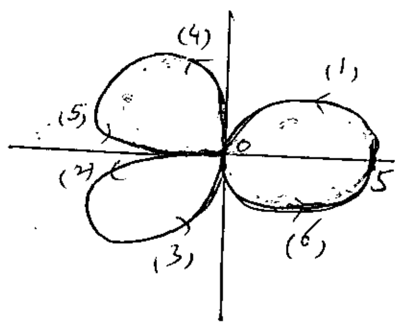
$r = 5\cos 3(\theta+\pi) = -5\cos 3\theta \Rightarrow$ " " not symmetric " origin

$r = 5\cos 3(\pi-\theta) = -5\cos 3\theta \Rightarrow$ " " " " " y-axis.

(b) Use $\theta = \pi/6$ as a scale to draw the graph of this curve.

	θ	r
(1)	$0 \to \pi/6$	$5 \to 0$
(2)	$\pi/6 \to \pi/3$	$0 \to -5$
(3)	$\pi/3 \to \pi/2$	$-5 \to 0$
(4)	$\pi/2 \to 2\pi/3$	$0 \to 5$
(5)	$2\pi/3 \to 5\pi/6$	$5 \to 0$
(6)	$5\pi/6 \to \pi$	$0 \to -5$

$r = 5\cos 3\theta$. Here $n = 3$ (odd) so the rose has 3 loops through origin with radius 5.



2. Change the polar equation $r = \sec\theta - \csc\theta$ to a Cartesian equation.

$$x = r\cos\theta = (\sec\theta - \csc\theta)\cos\theta$$

$$= \left(\frac{1}{\cos\theta} - \frac{1}{\sin\theta}\right)\cos\theta$$

$$= 1 - \cot\theta$$

$$= 1 - \frac{x}{y} \qquad \left(\because \tan\theta = \frac{y}{x} \Rightarrow \cot\theta = \frac{x}{y}\right)$$

$$x = \frac{y-x}{y} \Rightarrow xy = y - x$$

$$xy - y = -x$$

$$y - xy = x$$

$$y(1-x) = x$$

$$\Rightarrow y = \frac{x}{1-x}$$

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1. Find points on the polar curve $r = 1 - \cos\theta$ at which the tangent line is vertical.

$$x = r\cos\theta = (1-\cos\theta)\cos\theta = \cos\theta - \cos^2\theta$$

$$\frac{dx}{d\theta} = -\sin\theta + 2\cos\theta\sin\theta$$

put $\frac{dx}{d\theta}=0 \Rightarrow 0 = \sin\theta(2\cos\theta - 1) \Rightarrow \sin\theta = 0$ & $\cos\theta = \frac{1}{2}$

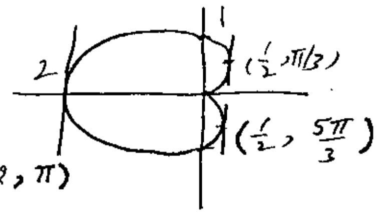
$$\Rightarrow \theta = 0, \pi/3, \pi, \frac{5\pi}{3}, 2\pi$$

$\theta = 0 \Rightarrow r = 0$; $(0,0)$

$\theta = \pi/3 \Rightarrow r = \frac{1}{2}$; $(\frac{1}{2}, \pi/3)$

$\theta = \pi \Rightarrow r = 2$; $(2, \pi)$

$\theta = \frac{5\pi}{3} \Rightarrow r = \frac{1}{2}$; $(\frac{1}{2}, \frac{5\pi}{3})$



2. Find values of t, for which the parametric curve $x = t^3 - 12\,t, y = t^2 - 7$ is concave upwards.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{3t^2-12}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{2t}{3t^2-12}\right)}{3t^2-12} = \frac{(3t^2-12)(2)-(2t)(6t)}{(3t^2-12)^3}$$

$$= \frac{-6t^2-24}{(3t^2-12)^3} = \frac{-6(t^2+4)}{3^3(t^2-4)} = \frac{\overset{-ve}{-2}\,\overset{\text{positive}}{(t^2+4)}}{9(t^2-4)}$$

Now $\frac{d^2y}{dx^2} > 0$ if $t^2-4<0 \Rightarrow t^2<4 \Rightarrow |t|^2<2^2 \Rightarrow |t|<2$
$\Rightarrow -2<t<2$

So the curve is concave upwards for $-2<t<2$.