Q1. (a) Sketch the polar curve $r = 2 + \cos 2\theta$. [**6pts**]

(b) Find the area enclosed by the curve given in (a). [**6pts**]

Q2. Consider the points $P(-1, 3, 1)$, $Q(0, 2, 3)$ and $R(-4, -1, -2)$.

(a) Find the area of the triangle PQR. [**6pts**]

(b) Find the scalar and vector projections of $\overrightarrow{PR}$ onto $\overrightarrow{PQ}$. [**6pts**]

Q3. Consider the function $f(x, y) = 2x^2 - 4xy + y^4$.

(a) Find all critical points of $f(x, y)$. [**4pts**]

(b) Find all second derivatives of $f(x, y)$. [**2pts**]

(c) Find the local maximum and minimum values and saddle points of $f(x, y)$, if any. [**5pts**]

Q4. Evaluate the iterated integral.

(**a**) $\displaystyle\int_0^1 \int_0^z \int_0^{x+z} 6xz \, dydxdz$ [**6pts**]

(**b**) $\displaystyle\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dydx$ [**7pts**]

Q5. Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3$. [**10pts**]

Q6. Use spherical coordinates to evaluate

$$\iiint_E xyz \, dV,$$

where E lies between the spheres $\rho = 1$ and $\rho = 2$ and above the cone $\phi = \pi/3$ in the first octant. [**10pts**]

Q1. The tangent line to the parametric curve $x = 1 + \ln t,\ y = t^2 + 2$ at the point $(1, 3)$ contains the point

(A) $(-1, 1)$

(B) $(0, 0)$

(C) $(0, -1)$

(D) $(-1, 0)$

(E) $(0, 1)$

Q2. Let E be a solid bounded by the planes $x = 0,\ y = 0,\ z = 0$ and $2x+2y+z = 2$. The triple integral

$$\iiint_E y\ dV$$

is equal to

(A) $\frac{1}{24}$

(B) $\frac{1}{6}$

(C) $\frac{1}{12}$

(D) $\frac{1}{4}$

(E) $\frac{1}{2}$

Q3. If $z = xy + xe^{y/x}$, then $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$ is equal to

(A) xy

(B) $xy + z$

(C) xyz

(D) $xz + y$

(E) $x + yz$

Q4. The volume of the solid bounded by the surfaces $z = x^2 + y^2$ and $z = 12 - 2x^2 - 2y^2$ is equal to

(A) $\displaystyle\int_0^4 \int_0^{\pi} \int_{r^2}^{12-2r^2} r \; dz d\theta dr$

(B) $\displaystyle\int_0^2 \int_0^{2\pi} \int_{2r^2}^{12-r^2} r \; dz d\theta dr$

(C) $\displaystyle\int_0^4 \int_0^{2\pi} \int_{r^2}^{12-2r^2} r \; dz d\theta dr$

(D) $\displaystyle\int_0^2 \int_0^{\pi} \int_{r^2}^{12-2r^2} r \; dz d\theta dr$

(E) $\displaystyle\int_0^2 \int_0^{2\pi} \int_{r^2}^{12-2r^2} r \; dz d\theta dr$

Q5. Let A and B be the maximum and minimum values of $f(x, y, z) = x + 2y - 3z$ subject to the constraint $x^2 + 2y^2 + 6z^2 = 2$, respectively. The value of $|A| + |B|$ is equal to

(A) 3

(B) 2

(C) $2\sqrt{3}$

(D) 6

(E) 1

Q6. Using the linear approximation of

$$f(x, y) = \ln\left(\frac{x^2}{y+1}\right)$$

at the point $(-1, 0)$, the value of $f(-0.96, 0.11)$ is approximately equal to

(A) 0.18

(B) -0.19

(C) -1

(D) 0.08

(E) -0.94

Q7. The arc length of the polar curve $r = 3 + 3\cos\theta,\ 0 \le \theta \le \pi/3$ is equal to

(A) 6

(B) 1

(C) 2

(D) 12

(E) 3

Q8. Let

$$G(x,y) = \begin{cases} \dfrac{2-\sqrt{4-x^2-y^2}}{x^2+y^2}, & (x,y) \ne (0,0) \\ a, & (x,y) = (0,0). \end{cases}$$

What is the value of a such that G is continuous at the point $(0,0)$?

(A) 2

(B) 4

(C) $\frac{1}{2}$

(D) 0

(E) $\frac{1}{4}$

Q9. The distance between the two planes $x + 2y - 2z = -1$ and $2x + 4y - 4z = 28$ is equal to

(A) 5

(B) 6

(C) 0

(D) 1

(E) 2

Q10. The directional derivative of $f(x, y, z) = x^2y + x\sqrt{1 + z}$ at the point $(2, -1, 3)$ in the direction of $\vec{v} = 2\vec{i} + \vec{j} - 2\vec{k}$ is equal to

(A) -2

(B) -1

(C) $-\dfrac{4}{3}$

(D) $-\dfrac{1}{3}$

(E) $-\dfrac{2}{3}$

Q11. Let D be the triangular region on the xy-plane with vertices $(0,0)$, $(1,1)$ and $(0,1)$. The double integral

$$\iint_D \frac{1}{1+x^2}\,dA$$

is equal to

(A) $\displaystyle\int_0^1 \int_y^1 \frac{1}{1+x^2}\,dxdy$

(B) $\displaystyle\int_0^1 \int_x^1 \frac{1}{1+x^2}\,dydx$

(C) $\displaystyle\int_0^1 \int_0^1 \frac{1}{1+x^2}\,dydx$

(D) $\displaystyle\int_0^1 \int_1^y \frac{1}{1+x^2}\,dxdy$

(E) $\displaystyle\int_0^1 \int_0^x \frac{1}{1+x^2}\,dydx$

Q12. The iterated integral

$$\int_0^1 \int_{\sqrt{y}}^1 3e^{x^3}\,dxdy$$

is equal to (Hint: *reverse the order of integration*)

(A) $e+1$

(B) e

(C) $e-1$

(D) $2e$

(E) $2e+1$

Q1. The arc length of the polar curve $r = 2 + 2\cos\theta,\ 0 \le \theta \le \pi/3$ is equal to

(A) 2

(B) 4

(C) 1

(D) 8

(E) 3

Q2. Using the linear approximation of

$$f(x, y) = \ln\left(\frac{x^2}{y+1}\right)$$

at the point $(-1, 0)$, the value of $f(-0.97, 0.12)$ is approximately equal to

(A) -0.95

(B) 0.19

(C) -1

(D) 0.06

(E) -0.18

Q3. Let

$$G(x,y) = \begin{cases} \dfrac{2-\sqrt{4-x^2-y^2}}{2x^2+2y^2}, & (x,y) \neq (0,0) \\ a, & (x,y) = (0,0). \end{cases}$$

What is the value of a such that G is continuous at the point $(0,0)$?

(A) 8

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 0

(E) $\frac{1}{8}$

Q4. Let D be the triangular region on the xy-plane with vertices $(0,0)$, $(1,1)$ and $(1,0)$. The double integral

$$\iint_D \frac{1}{1+y^2}\, dA$$

is equal to

(A) $\displaystyle\int_0^1 \int_y^1 \frac{1}{1+y^2}\, dxdy$

(B) $\displaystyle\int_0^1 \int_x^1 \frac{1}{1+y^2}\, dydx$

(C) $\displaystyle\int_0^1 \int_0^1 \frac{1}{1+y^2}\, dydx$

(D) $\displaystyle\int_0^1 \int_1^x \frac{1}{1+y^2}\, dydx$

(E) $\displaystyle\int_0^1 \int_0^y \frac{1}{1+y^2}\, dxdy$

Q5. The volume of the solid bounded by the surfaces $z = 2x^2 + 2y^2$ and $z = 9 - x^2 - y^2$ is equal to

(A) $\int_0^{\sqrt{3}} \int_0^{2\pi} \int_{2r^2}^{9-r^2} r \; dz d\theta dr$

(B) $\int_0^{\sqrt{3}} \int_0^{\pi} \int_{2r^2}^{9-r^2} r \; dz d\theta dr$

(C) $\int_0^{3} \int_0^{2\pi} \int_{2r^2}^{9-r^2} r \; dz d\theta dr$

(D) $\int_0^{3} \int_0^{\pi} \int_{2r^2}^{9-r^2} r \; dz d\theta dr$

(E) $\int_0^{\sqrt{3}} \int_0^{2\pi} \int_{r^2}^{9-2r^2} r \; dz d\theta dr$

Q6. The tangent line to the parametric curve $x = 2 + \ln t,\; y = t^2 + 1$ at the point $(2, 2)$ contains the point

(A) $(-2, -2)$

(B) $(-1, 0)$

(C) $(0, -1)$

(D) $(0, -2)$

(E) $(0, 0)$

Q7. Let A and B be the maximum and minimum values of $f(x, y, z) = x + 2y - 3z$ subject to the constraint $x^2 + 2y^2 + 6z^2 = 2$, respectively. The value of $|A| + |B|$ is equal to

(A) 3

(B) 6

(C) 1

(D) 2

(E) $2\sqrt{3}$

Q8. The distance between the two planes $x + 2y - 2z = -1$ and $2x + 4y - 4z = 16$ is equal to

(A) 0

(B) 6

(C) 3

(D) 1

(E) 4

Q9. The directional derivative of $f(x, y, z) = x^2y + x\sqrt{1+z}$ at the point $(3, -1, 3)$ in the direction of $\vec{v} = 2\vec{i} + \vec{j} - 2\vec{k}$ is equal to

(A) $-\frac{1}{2}$

(B) $-\frac{1}{3}$

(C) $-\frac{5}{6}$

(D) $-\frac{1}{6}$

(E) $-\frac{2}{3}$

Q10. The iterated integral

$$\int_0^4 \int_{\sqrt{x}}^2 3e^{y^3}\, dydx$$

is equal to (Hint: *reverse the order of integration*)

(A) e^8

(B) $e^8 - 1$

(C) $e^8 + 1$

(D) $2e^8$

(E) $2e^8 + 1$

Q11. Let E be a solid bounded by the planes $x = 0,\ y = 0,\ z = 0$ and $2x+2y+z = 2$. The triple integral

$$\iiint\limits_E 2y\ dV$$

is equal to

(A) $\frac{1}{2}$

(B) $\frac{1}{12}$

(C) $\frac{1}{24}$

(D) $\frac{1}{4}$

(E) $\frac{1}{6}$

Q12. If $z = 2xy + xe^{y/x}$, then $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$ is equal to

(A) $2xyz$

(B) $2xy$

(C) $2xy + z$

(D) $2xz + y$

(E) $2x + yz$

Q1. Let D be the triangular region on the xy-plane with vertices $(0,0)$, $(1,1)$ and $(1,0)$. The double integral

$$\iint_D \frac{1}{1+y^2}\,dA$$

is equal to

(A) $\displaystyle\int_0^1\int_0^y \frac{1}{1+y^2}\,dxdy$

(B) $\displaystyle\int_0^1\int_x^1 \frac{1}{1+y^2}\,dydx$

(C) $\displaystyle\int_0^1\int_y^1 \frac{1}{1+y^2}\,dxdy$

(D) $\displaystyle\int_0^1\int_1^x \frac{1}{1+y^2}\,dydx$

(E) $\displaystyle\int_0^1\int_0^1 \frac{1}{1+y^2}\,dydx$

Q2. The volume of the solid bounded by the surfaces $z = x^2+y^2$ and $z = 12-2x^2-2y^2$ is equal to

(A) $\displaystyle\int_0^4\int_0^{\pi}\int_{r^2}^{12-2r^2} r\ dzd\theta dr$

(B) $\displaystyle\int_0^2\int_0^{\pi}\int_{r^2}^{12-2r^2} r\ dzd\theta dr$

(C) $\displaystyle\int_0^4\int_0^{2\pi}\int_{r^2}^{12-2r^2} r\ dzd\theta dr$

(D) $\displaystyle\int_0^2\int_0^{2\pi}\int_{r^2}^{12-2r^2} r\ dzd\theta dr$

(E) $\displaystyle\int_0^2\int_0^{2\pi}\int_{2r^2}^{12-r^2} r\ dzd\theta dr$

Q3. The iterated integral

$$\int_0^4 \int_{\sqrt{x}}^2 3e^{y^3}\, dydx$$

is equal to (Hint: *reverse the order of integration*)

(A) $e^8 - 1$

(B) $2e^8 + 1$

(C) e^8

(D) $2e^8$

(E) $e^8 + 1$

Q4. If $z = 2xy + xe^{y/x}$, then $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$ is equal to

(A) $2x + yz$

(B) $2xy$

(C) $2xy + z$

(D) $2xz + y$

(E) $2xyz$

Q5. Let E be a solid bounded by the planes $x = 0,\ y = 0,\ z = 0$ and $2x+2y+z = 2$. The triple integral

$$\iiint_E 2y\ dV$$

is equal to

(A) $\frac{1}{6}$

(B) $\frac{1}{24}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

(E) $\frac{1}{12}$

Q6. The arc length of the polar curve $r = 3 + 3\cos\theta,\ 0 \le \theta \le \pi/3$ is equal to

(A) 2

(B) 1

(C) 12

(D) 3

(E) 6

Q7. The directional derivative of $f(x,y,z) = x^2y + x\sqrt{1+z}$ at the point $(2,-1,3)$ in the direction of $\vec{v} = 2\vec{i} + \vec{j} - 2\vec{k}$ is equal to

(A) $-\frac{2}{3}$

(B) $-\frac{1}{3}$

(C) $-\frac{4}{3}$

(D) -1

(E) -2

Q8. Let

$$G(x,y) = \begin{cases} \dfrac{2-\sqrt{4-x^2-y^2}}{2x^2+2y^2}, & (x,y) \neq (0,0) \\ a, & (x,y) = (0,0). \end{cases}$$

What is the value of a such that G is continuous at the point $(0,0)$?

(A) 0

(B) $\frac{1}{8}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

(E) 8

Q9. The tangent line to the parametric curve $x = 2 + \ln t,\ y = t^2 + 1$ at the point $(2, 2)$ contains the point

(A) $(0, -1)$

(B) $(0, 0)$

(C) $(-1, 0)$

(D) $(-2, -2)$

(E) $(0, -2)$

Q10. The distance between the two planes $x + 2y - 2z = -1$ and $2x + 4y - 4z = 28$ is equal to

(A) 5

(B) 0

(C) 1

(D) 6

(E) 2

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at the point $(-1,0)$, the value of $f(-0.96, 0.11)$ is approximately equal to

(A) -1

(B) 0.18

(C) -0.94

(D) 0.08

(E) -0.19

Q12. Let A and B be the maximum and minimum values of $f(x,y,z) = x+2y-3z$ subject to the constraint $x^2+2y^2+6z^2 = 2$, respectively. The value of $|A|+|B|$ is equal to

(A) 3

(B) 6

(C) $2\sqrt{3}$

(D) 2

(E) 1

Q1. The iterated integral

$$\int_0^1 \int_{\sqrt{y}}^1 3e^{x^3}\,dxdy$$

is equal to (Hint: *reverse the order of integration*)

(A) $2e + 1$

(B) e

(C) $e + 1$

(D) $e - 1$

(E) $2e$

Q2. Using the linear approximation of

$$f(x, y) = \ln\left(\frac{x^2}{y+1}\right)$$

at the point $(-1, 0)$, the value of $f(-0.97, 0.12)$ is approximately equal to

(A) 0.19

(B) -0.95

(C) -1

(D) 0.06

(E) -0.18

Q3. The directional derivative of $f(x, y, z) = x^2y + x\sqrt{1 + z}$ at the point $(3, -1, 3)$ in the direction of $\vec{v} = 2\vec{i} + \vec{j} - 2\vec{k}$ is equal to

(A) $-\frac{5}{6}$

(B) $-\frac{1}{3}$

(C) $-\frac{2}{3}$

(D) $-\frac{1}{2}$

(E) $-\frac{1}{6}$

Q4. The arc length of the polar curve $r = 2 + 2\cos\theta,\ 0 \le \theta \le \pi/3$ is equal to

(A) 4

(B) 3

(C) 2

(D) 1

(E) 8

Q5. The tangent line to the parametric curve $x = 1 + \ln t,\ y = t^2 + 2$ at the point $(1, 3)$ contains the point

(A) $(0, 1)$

(B) $(-1, 0)$

(C) $(0, -1)$

(D) $(-1, 1)$

(E) $(0, 0)$

Q6. Let

$$G(x,y) = \begin{cases} \dfrac{2 - \sqrt{4 - x^2 - y^2}}{x^2 + y^2}, & (x,y) \neq (0,0) \\ a, & (x,y) = (0,0). \end{cases}$$

What is the value of a such that G is continuous at the point $(0, 0)$?

(A) $\dfrac{1}{2}$

(B) 4

(C) 2

(D) $\dfrac{1}{4}$

(E) 0

Q7. The volume of the solid bounded by the surfaces $z = 2x^2 + 2y^2$ and $z = 9 - x^2 - y^2$ is equal to

(A) $\int_0^3 \int_0^{\pi} \int_{2r^2}^{9-r^2} r \, dz d\theta dr$

(B) $\int_0^{\sqrt{3}} \int_0^{\pi} \int_{2r^2}^{9-r^2} r \, dz d\theta dr$

(C) $\int_0^{\sqrt{3}} \int_0^{2\pi} \int_{2r^2}^{9-r^2} r \, dz d\theta dr$

(D) $\int_0^{\sqrt{3}} \int_0^{2\pi} \int_{r^2}^{9-2r^2} r \, dz d\theta dr$

(E) $\int_0^3 \int_0^{2\pi} \int_{2r^2}^{9-r^2} r \, dz d\theta dr$

Q8. The distance between the two planes $x + 2y - 2z = -1$ and $2x + 4y - 4z = 16$ is equal to

(A) 6

(B) 3

(C) 4

(D) 0

(E) 1

Q9. If $z = xy + xe^{y/x}$, then $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$ is equal to

(A) $x + yz$

(B) xy

(C) $xz + y$

(D) $xy + z$

(E) xyz

Q10. Let A and B be the maximum and minimum values of $f(x, y, z) = x + 2y - 3z$ subject to the constraint $x^2 + 2y^2 + 6z^2 = 2$, respectively. The value of $|A| + |B|$ is equal to

(A) 3

(B) 6

(C) 2

(D) $2\sqrt{3}$

(E) 1

Q11. Let D be the triangular region on the xy-plane with vertices $(0,0)$, $(1,1)$ and $(0,1)$. The double integral

$$\iint\limits_D \frac{1}{1+x^2}\, dA$$

is equal to

(A) $\displaystyle\int_0^1 \int_1^y \frac{1}{1+x^2}\, dxdy$

(B) $\displaystyle\int_0^1 \int_y^1 \frac{1}{1+x^2}\, dxdy$

(C) $\displaystyle\int_0^1 \int_x^1 \frac{1}{1+x^2}\, dydx$

(D) $\displaystyle\int_0^1 \int_0^x \frac{1}{1+x^2}\, dydx$

(E) $\displaystyle\int_0^1 \int_0^1 \frac{1}{1+x^2}\, dydx$

Q12. Let E be a solid bounded by the planes $x=0,\ y=0,\ z=0$ and $2x+2y+z=2$. The triple integral

$$\iiint\limits_E y\, dV$$

is equal to

(A) $\frac{1}{24}$

(B) $\frac{1}{12}$

(C) $\frac{1}{6}$

(D) $\frac{1}{4}$

(E) $\frac{1}{2}$