Q1. (a) Find the point at which the given lines intersect: **[5pts]**

$$L_1: \quad x = t,\ y = 2 - t,\ z = -2 + 2t$$
$$L_2: \quad x = 3 - s,\ y = -1 + s,\ z = -2 + s$$

(b) Find an equation of the plane that contains the lines L_1 and L_2. **[7pts]**

Q2. Find and sketch the domain of the function $f(x, y) = \dfrac{1}{\ln(xy)}$. **[5pts]**

Q3. Find the linear approximation $L(x, y)$ of the function $f(x, y) = x^2 y + \sqrt{x^2 + y^2}$ at the point $(1, 0)$ and use it to approximate the value of $f(0.98, 0.03)$. **[10pts]**

Q4. (a) Let $z = \tan^{-1}\left(\dfrac{x}{y^2}\right)$. Find $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y}$ at the point $(x, y) = (-1, -1)$. **[5pts]**

(b) Let $F(x, y) = x\cos(y) + \sin(xy)$. Find $F_{xyx}(1, \pi/2)$. **[6pts]**

Q5. Find the absolute maximum and minimum values of $f(x, y) = e^{x^2 y}$ on the closed triangular region with vertices $(0, -1)$, $(0, 2)$ and $(1, -1)$. **[13pts]**

Q1. Consider the surface

$$x^2z + 3yz^2 + 3xyz = 7.$$

Let $5x + By + Cz = D$ be an equation of the tangent plane to the given surface at $(1, 1, 1)$. The value of $B + C + D$ is equal to

(A) 39

(B) 32

(C) 42

(D) 36

(E) 37

Q2. The function $f(x, y) = 2x^3y + 3x^2 + y^2$ has

(A) one saddle point and two local maxima

(B) one local maximum and two saddle points

(C) two local minima and one local maximum

(D) one local minimum and two saddle points

(E) three saddle points

Q3. Let $T(x, y) = e^{x^2-y^2}$. The function T increases most rapidly at the point $(1, 1)$ in the direction of the vector

(A) $\langle 1, -2\rangle$

(B) $\langle 1, -1\rangle$

(C) $\langle -2, 1\rangle$

(D) $\langle -1, 1\rangle$

(E) $\langle 1, 1\rangle$

Q4. Let $W(s, t) = F(u(s, t), v(s, t))$, where F, u and v are differentiable functions and

$$u(1,0) = 2, \quad v(1,0) = -3, \quad u_s(1,0) = -2, \quad v_s(1,0) = 5,$$
$$u_t(1,0) = -6, \quad v_t(1,0) = 4, \quad F_u(2,-3) = 1, \quad F_v(2,-3) = 3.$$

The value of $W_s(1, 0) + W_t(1, 0)$ is equal to

(A) 19

(B) 22

(C) 21

(D) 20

(E) 18

Q5. $\lim_{(x,y)\to(0,0)} \dfrac{1-\cos(3x^2+3y^2)}{\sin^2(x^2+y^2)}$ is equal to

(A) $\dfrac{9}{2}$

(B) 3

(C) $\dfrac{1}{2}$

(D) 2

(E) $\dfrac{3}{2}$

Q6. Symmetric equations for the line of intersection of the planes $z = 2x - y - 5$ and $z = x + 3y - 1$ are

(A) $\dfrac{x-4}{-2} = y = \dfrac{z-3}{7}$

(B) $\dfrac{x}{4} = y + 2 = \dfrac{z+3}{7}$

(C) $\dfrac{x-4}{4} = y = \dfrac{z-3}{7}$

(D) $\dfrac{x}{-2} = y + 2 = \dfrac{z+3}{7}$

(E) $\dfrac{x-4}{4} = -y = \dfrac{z-3}{7}$

Q7. The equation $-2x^2 - y^2 + z^2 - 2y - 4z - 12x + 5 = 0$ represents

(A) an ellipsoid

(B) a hyperboloid of one sheet

(C) a hyperbolic paraboloid

(D) a cone

(E) a hyperboloid of two sheets

Q1. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{\sin^2(2x^2+2y^2)}$ is equal to

(A) $\frac{1}{8}$

(B) $\frac{1}{2}$

(C) 2

(D) 1

(E) $\frac{1}{4}$

Q2. Symmetric equations for the line of intersection of the planes $z = 2x - y - 5$ and $z = x + 3y - 4$ are

(A) $\frac{x-1}{4} = -y = \frac{z+3}{7}$

(B) $\frac{x}{4} = y + 2 = \frac{z+3}{7}$

(C) $\frac{x-1}{-2} = y = \frac{z+3}{7}$

(D) $\frac{x-1}{4} = y = \frac{z+3}{7}$

(E) $\frac{x}{4} = y + 2 = \frac{z+3}{5}$

Q3. The function $f(x,y) = 2x^3y - 3x^2 - y^2$ has

(A) two local minima and one local maximum

(B) one saddle point and two local maxima

(C) one local maximum and two saddle points

(D) three saddle points

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The value of $W_s(1,0) + W_t(1,0)$ is equal to

(A) -16

(B) -13

(C) -15

(D) -12

(E) -14

Q5. Consider the surface

$$x^2z + 3yz^2 + 3xyz = 7.$$

Let $Ax + 6y + Cz = D$ be an equation of the tangent plane to the given surface at $(1, 1, 1)$. The value of $A + C + D$ is equal to

(A) 32

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(C) 42

(D) 37

(E) 39

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(E) -16

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Let $5x + By + Cz = D$ be an equation of the tangent plane to the given surface at $(1, 1, 1)$. The value of $B + C + D$ is equal to

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(D) $\dfrac{x}{4} = y + 2 = \dfrac{z+3}{5}$

(E) $\dfrac{x-1}{-2} = y = \dfrac{z+3}{7}$