Q1. (a) Find the point at which the given lines intersect: **[5pts]**

$$L_1: \quad x = t,\ y = 2 - t,\ z = -2 + 2t$$
$$L_2: \quad x = 3 - s,\ y = -1 + s,\ z = -2 + s$$

Solution:

Solving sytem of equations

$$\begin{cases} t = 3 - s \\ 2 - t = -1 + s \\ -2 + 2t = -2 + s \end{cases}$$ 2

gives $t = 1$ and $s = 2$ 2

and $x = 1,\ y = 1,\ z = 0.$

The intersection point of L_1 and L_1 1

is $(1, 1, 0)$

(b) Find an equation of the plane that contains the lines L_1 and L_2. **[7pts]**

Solution:

$$\vec{V_1} = \langle 1, -1, 2 \rangle$$
$$\vec{V_2} = \langle -1, 1, 1 \rangle$$ 1 + 1

The normal vector of the plane is

$$\vec{n} = \vec{V_1} \times \vec{V_2}$$ 1

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ -1 & 1 & 1 \end{vmatrix}$$

$$= -3\vec{i} - 3\vec{j}$$ 2

An equation of the plane is

$$-3(x-1) - 3(y-1) = 0$$

$$\Longleftrightarrow \quad 3x + 3y - 6 = 0$$

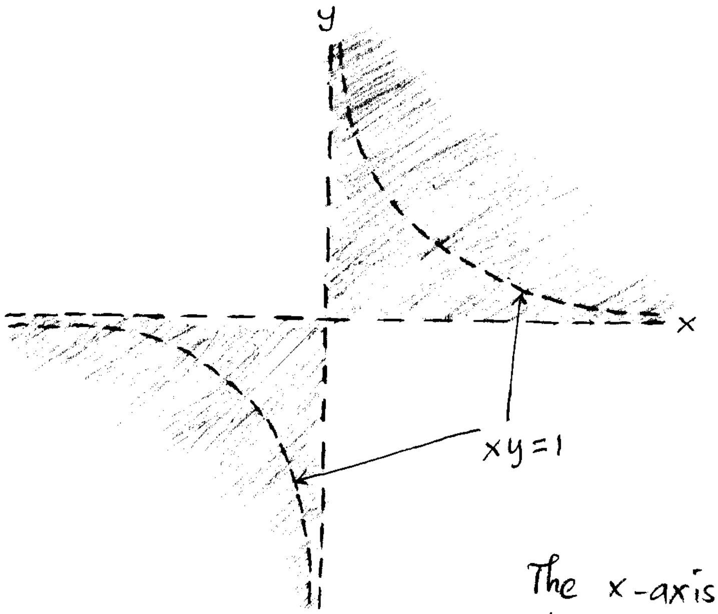
$$\Longleftrightarrow \quad x + y = 2$$ 2

Q2. Find and sketch the domain of the function $f(x,y) = \dfrac{1}{\ln(xy)}$. **[5pts]**

Solution:

The domain is

$$D = \{(x,y) \mid xy > 0,\ xy \neq 1\} \qquad 2$$



3

The x-axis, The y-axis, The line $xy=1$ are NOT included in the domain (dashed lines)

Q3. Find the linear approximation $L(x,y)$ of the function $f(x,y) = x^2y + \sqrt{x^2+y^2}$ at the point $(1,0)$ and use it to approximate the value of $f(0.98, 0.03)$. **[10pts]**

Solution: $f(1,0) = 1$ 1

$f_x(x,y) = 2xy + \dfrac{x}{\sqrt{x^2+y^2}}$, $f_x(1,0) = 1$ 2

$f_y(x,y) = x^2 + \dfrac{y}{\sqrt{x^2+y^2}}$, $f_y(1,0) = 1$ 2

The linear approximation of f at $(1,0)$ is

$$L(x,y) = f(1,0) + f_x(1,0)(x-1) + f_y(1,0)\,y$$
$$= 1 + (x+1) + y = x + y \qquad 3$$

$$f(0.98, 0.03) \approx L(0.98, 0.03) = 0.98 + 0.03$$
$$= 1.01 \qquad 2$$

Q4. (a) Let $z = \tan^{-1}\left(\frac{x}{y^2}\right)$. Find $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}$ at the point $(x, y) = (-1, -1)$. **[5pts]**

Solution:

$$\frac{\partial z}{\partial x} = \left(\frac{1}{1+\left(\frac{x}{y^2}\right)^2}\right)\left(\frac{1}{y^2}\right) = \frac{y^2}{y^4+x^2} \qquad 2$$

$$\frac{\partial z}{\partial y} = \left(\frac{1}{1+\left(\frac{x}{y^2}\right)^2}\right)\left(\frac{-2x}{y^3}\right) = -\frac{2xy}{y^4+x^2} \qquad 2$$

$$\left(\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}\right)\Bigg|_{(x,y)=(-1,-1)} = \frac{1}{2} - 1 = -\frac{1}{2} \qquad 1$$

(b) Let $F(x, y) = x\cos(y) + \sin(xy)$. Find $F_{xyx}(1, \pi/2)$. **[6pts]**

Solution:

$$F_x(x,y) = \cos y + y\cos xy \qquad 1$$

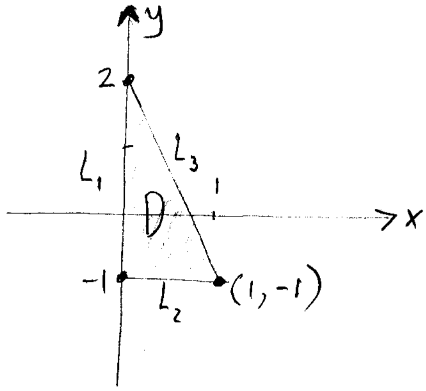
$$F_{xy}(x,y) = -\sin y + \cos xy - xy\sin xy \qquad 2$$

$$\left.\begin{aligned} F_{xyx}(x,y) &= -y\sin xy - y\sin xy - xy^2\cos xy \\ &= -2y\sin xy - xy^2\cos xy \end{aligned}\right] \qquad 2$$

$$F_{xyx}(1, \pi/2) = -\pi \qquad 1$$

Q5. Find the absolute maximum and minimum values of $f(x, y) = e^{x^2 y}$ on the closed triangular region with vertices $(0, -1)$, $(0, 2)$ and $(1, -1)$. **[13pts]**

Solution



⊙ Critical points

(2) $\begin{cases} f_x = 2xye^{x^2y} = 0 \\ f_y = x^2 e^{x^2y} = 0 \end{cases}$

$\Rightarrow x = 0$ is not in the interior of D.

(no critical point in the interior of D)

⊙ Boundary of D

(2) L_1: $x = 0$, $-1 \le y \le 2$

$f(0,y) = e^0 = 1$

L_2: $y = -1$, $0 \le x \le 1$

$f(x,-1) = e^{-x^2} \equiv g(x)$

$g'(x) = -2x e^{-x^2} = 0 \iff x = 0$

$f(0,-1) = e^0 = 1$

(1) End point $\Rightarrow$ $f(1,-1) = e^{-1}$

L_3: $y = -3x + 2$, $0 \le x \le 1$

$f(x,y) = e^{x^2(-3x+2)} = e^{-3x^3 + 2x^2} \equiv h(x)$ (2)

$h'(x) = -9x^2 + 4x = 0$

$\iff x(-9x+4) = 0$

$\iff x = 0$ or $x = \frac{4}{9}$ (2)

$\Updownarrow \qquad \Updownarrow$

$y = 2 \qquad y = -\frac{4}{3} + 2 = \frac{2}{3}$

(1) + (1) $f(0,2) = e^0 = 1$

$f\left(\frac{4}{9}, \frac{2}{3}\right) = e^{\left(\frac{4}{9}\right)^2\left(\frac{2}{3}\right)}$

$= e^{32/243}$

The abs maximum is $f\left(\frac{4}{9}, \frac{2}{3}\right) = e^{\frac{32}{243}}$

The abs minimum is $f(1,-1) = e^{-1}$ (1) + (1)