King Fahd University of Petroleum and Minerals
Quiz 1 Math 102-112 Duration 25 minutes

Question 1 Use three rectangles and midpoints to approximate the area under the graph of $f(x) = \frac{x}{x+2} + 1$ from $x = -1$ to $x = 2$.
Solution

$$\Delta x = \frac{2-(-1)}{3} = 1,$$

and hence, we can estimate the area A as:

$$A \approx \Delta x\,(f(-1/2) + f(1/2) + f(3/2))$$
$$= (\frac{-1}{3} + 1) + (\frac{1}{5} + 1) + (\frac{3}{7} + 1) = \frac{31}{105} + 3 = \frac{346}{105}.$$

Question 2 Evaluate

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{2n-i+1}.$$

Solution

$$\sum_{i=1}^{n} \frac{1}{2n-i+1} = \sum_{i=1}^{n} \frac{1}{n} \left(\frac{1}{2 - \frac{(i-1)}{n}} \right)$$

Let $x_{i-1} = \frac{i-1}{n}$ and so, $a = 0$ and $\Delta x = \frac{1}{n}$. Then, $1 = b - a \Longrightarrow b = 1$. Therefore,

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{2n-i+1} = \int_0^1 \frac{1}{2-x}\,dx = -\ln|2-x| \Big|_0^1 = \ln(2).$$

Question 3 Evaluate

$$\int_{-2}^{1} \sqrt{8 - x^2 + 2x}\,dx.$$

(You may interpreting it as an area.)
Solution Completing square

$$\int_{-2}^{1} \sqrt{8 - x^2 + 2x}\,dx = \int_{-2}^{1} \sqrt{9 - (x-1)^2}\,dx = \frac{\pi(3)^2}{4} = \frac{9\pi}{4}$$

Question 4 Evaluate $f(\mathrm{e})$ if

$$\int_{\mathrm{e}^{x^2}}^{2} \ln(t) f(t)\,dt = \mathrm{e}x - 3f(1).$$

Solution Differentiate both sides and using the FTCI, we get

$$-\ln(\mathrm{e}^{x^2}) f(\mathrm{e}^{x^2}) 2x\mathrm{e}^{x^2} = \mathrm{e} \Longrightarrow -2x^3\mathrm{e}^{x^2} f(\mathrm{e}^{x^2}) = \mathrm{e}.$$

Substitute $x = 1$, we obtain

$$-2\mathrm{e}f(\mathrm{e}) = \mathrm{e} \Longrightarrow f(\mathrm{e}) = -1/2.$$

Question 5 If the velocity of a particle moving along a straight line is given by $v(t) = 1 - 2\sin t$, then find the distance traveled during the interval time $[0, \frac{\pi}{2}]$.
Solution Let d be the distance traveled during the time interval $[0, \frac{\pi}{2}]$. So,

$$d = \int_0^{\frac{\pi}{2}} |v(t)|\,dt = \int_0^{\frac{\pi}{6}} v(t)\,dt + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (-v(t))\,dt$$

$$= \int_0^{\frac{\pi}{6}} (1 - 2\sin t)\,dt - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 - 2\sin t)\,dt$$

$$= (t + 2\cos t)\Big|_0^{\frac{\pi}{6}} - (t + 2\cos t)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= (\frac{\pi}{6} + 2\cos(\frac{\pi}{6})) - (0 + 2\cos(0)) - (\frac{\pi}{2} + 2\cos(\frac{\pi}{2})) + (\frac{\pi}{6} + 2\cos(\frac{\pi}{6}))$$

$$= 2\sqrt{3} - \frac{\pi}{6} - 2.$$