King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Term 112
Monday 21/05/2012
Net Time Allowed: 180 minutes

MASTER VERSION

1. The series $\sum_{n=1}^{\infty} \frac{n}{5n-1}$

(a) is divergent

(b) converges by the integral test

(c) converges by the limit comparison test

(d) converges to $\frac{1}{5}$

(e) diverges by the ratio test

2. $\int \frac{e^{\sqrt{\sin x}} \cos x}{\sqrt{\sin x}} dx =$

(a) $2e^{\sqrt{\sin x}} + C$

(b) $-\frac{1}{2}e^{\sqrt{\sin x}} + C$

(c) $\frac{2e^{\sqrt{\sin x}}}{\sqrt{\sin x}} + C$

(d) $\frac{1}{4}e^{\sqrt{\sin x}} + C$

(e) $4e^{\sqrt{\sin x}} + C$

3. The sequence $\left\{\frac{4}{5}, \frac{6}{8}, \frac{8}{11}, \frac{10}{14}, \ldots\right\}$

(a) converges to $\frac{2}{3}$

(b) converges to $\frac{2}{5}$

(c) converges to 0

(d) converges to $\frac{1}{3}$

(e) is divergent

4. $\int_0^1 \frac{4^x + 1}{2^x} dx =$

(a) $\frac{3}{\ln 4}$

(b) $\frac{5}{\ln 2}$

(c) $\frac{5}{\ln 4}$

(d) $\frac{2}{\ln 2}$

(e) $\frac{1}{\ln 4}$

5. $\int_0^{\frac{\pi}{12}} (\cos x + \sin x)^2 \cos 2x \, dx =$

(a) $\frac{5}{16}$

(b) $\frac{9}{4}$

(c) $\frac{3}{16}$

(d) $\frac{3}{8}$

(e) $\frac{1}{16}$

6. The sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{3^{2n+1}}$ is

(a) $\frac{4}{33}$

(b) $\frac{4}{9}$

(c) $-\frac{4}{9}$

(d) $-\frac{2}{33}$

(e) $\frac{4}{11}$

7. The series $\sum_{n=1}^{\infty} \left(\frac{4 + 5\ln\sqrt{n}}{\sqrt{n} + 7} \right)^n$ is

(a) convergent

(b) a series with which the root test is not applicable

(c) divergent by the root test

(d) divergent with comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(e) divergent by the integral test.

8. If we use the Maclaurin series for $\sin x$, then the sum of the series $\frac{1}{6} - \frac{\pi}{6} + \frac{1}{3!}\left(\frac{\pi}{6}\right)^3 - \frac{1}{5!}\left(\frac{\pi}{6}\right)^5 + \ldots$ is

(a) $-\frac{1}{3}$

(b) $-\frac{1}{6}$

(c) $\frac{2}{3}$

(d) $\frac{1}{3}$

(e) $-\frac{2}{3}$

9. $\int x \sec^2(3x)\, dx =$

(a) $\frac{1}{3}x\tan(3x) - \frac{1}{9}\ln|\sec(3x)| + C$

(b) $3x\tan(3x) + \frac{1}{3}\ln|\sec(3x)| + C$

(c) $\frac{1}{6}x^2\tan(3x) + C$

(d) $\frac{1}{3}\tan(3x) + \frac{1}{2}x^2 + C$

(e) $\frac{1}{6}x\tan(3x) - \frac{1}{9}\ln|\sec x| + C$

10. The volume of the solid generated by rotating the region bounded by the graphs of $y = \cos x$ and $y = 0$ from $x = \frac{\pi}{2}$ to $x = \frac{3\pi}{2}$ about the line $x = \frac{\pi}{2}$ is equal to

(a) $2\pi^2$

(b) $\frac{3}{2}\pi^2$

(c) $4\pi^2$

(d) π^2

(e) $\frac{1}{2} + \pi^2$

11. $\int \frac{x}{\sqrt{8-2x-x^2}}\,dx =$

(a) $-\sin^{-1}\left(\frac{x+1}{3}\right) - \sqrt{8-2x-x^2} + C$

(b) $-\frac{1}{3}\sin^{-1}\left(\frac{x+1}{3}\right) - 3\sqrt{8-2x-x^2} + C$

(c) $-3\sin^{-1}\left(\frac{x+1}{3}\right) - \frac{1}{3}\sqrt{8-2x-x^2} + C$

(d) $-\frac{\sin^{-1}(x+1)}{\sqrt{8-2x-x^2}} + C$

(e) $-\sqrt{8-2x-x^2}\,\sin^{-1}(x+1) + C$

12. If $a_n = \frac{\cos^4(n^2+1)}{(n+1)^{3/2}}$, then the series $\sum_{n=1}^{\infty} a_n$ is

(a) convergent by the comparison test

(b) divergent by the comparison test

(c) convergent because $\lim_{n\to\infty} a_n = 0$

(d) divergent because $\lim_{n\to\infty} a_n \neq 0$

(e) a convergent geometric series.

13. $\int \frac{x+1}{x^3-x^2}\,dx =$

(a) $\frac{1}{x} + \ln\left(\frac{x-1}{x}\right)^2 + C$

(b) $-\frac{1}{x} + \ln\sqrt{\frac{x-1}{x}} + C$

(c) $\ln(x^2\sqrt{x-1}) + C$

(d) $\frac{1}{x} + \ln\left(\frac{x-1}{x}\right)^{3/2} + C$

(e) $\frac{1}{x} + \ln\left(\frac{x}{\sqrt{x-1}}\right) + C$

14. If $\{S_n\}$ is the sequence of partial sums of the series $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$, then $\lim_{n\to\infty} S_n$

(a) is equal to $\frac{3}{2}$

(b) is equal to 1

(c) is equal to $\frac{1}{2}$

(d) is equal to 0

(e) does not exist

15. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+2}-1}$

(a) converges conditionally

(b) converges absolutely

(c) diverges

(d) converges by the integral test

(e) diverges by the ratio test

16. The length of the curve $y = 3 - \ln \cos x$, $0 \leq x \leq \frac{\pi}{3}$, is

(a) $\ln(2+\sqrt{3})$

(b) $3+\ln(2+\sqrt{3})$

(c) $3-\ln(2+\sqrt{3})$

(d) $2+\ln 3$

(e) $\ln\sqrt{3}$

17. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-3)^n(x+1)^n}{\sqrt[3]{n}}$ is

(a) $\left(-\frac{4}{3}, -\frac{2}{3}\right]$

(b) $\left[-\frac{4}{3}, -\frac{2}{3}\right)$

(c) $\left(-\frac{4}{3}, -\frac{2}{3}\right)$

(d) $\left(-\frac{1}{3}, \frac{1}{3}\right]$

(e) $\left(-\frac{1}{3}, \frac{1}{3}\right)$

18. If $t = \tan \frac{x}{2}$, then $\int \frac{dx}{3 - 2\sin x + 5\cos x} =$

(a) $\int \frac{1}{4 - 2t - t^2} dt$

(b) $\int \frac{2}{6 - 5t + 7t^2} dt$

(c) $\int \frac{1}{8 - 2t + 4t^2} dt$

(d) $\int \frac{8}{4 - 2t - t^2} dt$

(e) $\int \frac{1}{4 + 2t - 3t^2} dt$

19. The area of the region bounded by the graphs of $y = \ln x,\ x + y = 1$, and the line $y = 1$, is equal to

(a) $e - \frac{3}{2}$

(b) $2\ln 2 - \frac{1}{2}$

(c) $2e - \frac{5}{2}$

(d) $1 - \ln 2$

(e) $e - 2$

20. The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n(n+1)}{3^n}$

(a) converges absolutely

(b) converges conditionally

(c) diverges by the test for divergence

(d) diverges by the ratio test

(e) converges by the integral test.

21. If the ratio test is applied to the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$, then the value of the limit L found by the ratio test

(a) is equal to $\frac{1}{e}$

(b) is equal to e

(c) is ∞

(d) is equal to 1

(e) is equal to 0.

22. The area of the surface generated by rotating the curve $y = \frac{1}{4}x^4 + \frac{1}{8x^2}$, $1 \le x \le 2$, about the y-axis, is given by the integral

(a) $\int_1^2 2\pi \left(x^4 + \frac{1}{4x^2} \right) dx$

(b) $\int_1^2 2\pi \left(x^4 + \frac{1}{x^4} \right) dx$

(c) $\int_1^2 2\pi \left(x^4 + \frac{1}{4x^3} \right) dx$

(d) $\int_1^2 2\pi \left(x^4 - \frac{1}{4x^4} \right) dx$

(e) $\int_1^2 2\pi \left(x^4 - \frac{1}{4x^3} \right) dx$

23. If we use the Maclaurin series for $f(x) = \dfrac{1}{1-2x}$, then the Maclanrin series for $g(x) = \dfrac{2}{(1-2x)^2}$ is [Hint : You may use differentiation]

(a) $\sum\limits_{n=0}^{\infty} 2^{n+1}(n+1)x^n$

(b) $\sum\limits_{n=0}^{\infty} 2^{n+2}(n+1)x^{n-1}$

(c) $\sum\limits_{n=0}^{\infty} 2^{n}(n+1)x^{n+1}$

(d) $\sum\limits_{n=0}^{\infty} 2(n+1)x^n$

(e) $\sum\limits_{n=0}^{\infty} 2nx^n$

24. $\displaystyle\int \frac{dx}{\sqrt{x}(\sqrt[4]{x}+1)} =$

(a) $4\sqrt[4]{x} - 4\ln(\sqrt[4]{x}+1) + C$

(b) $2\sqrt[4]{x} + 4\ln(\sqrt[4]{x}+1) + C$

(c) $4\sqrt[4]{x} - 2\ln(\sqrt[4]{x}+1) + C$

(d) $\ln\left(\dfrac{\sqrt[4]{x}+1}{\sqrt[4]{x}}\right)^2 + C$

(e) $\ln\left(\dfrac{\sqrt[4]{x}}{\sqrt[4]{x}+1}\right)^2 + C$

25. If k is a positive real number such that the improper integral $\int_1^\infty \frac{e^{x^k}}{x^{1-k}}\,dx$ is **divergent**, then

(a) k is any positive real number.

(b) $k > 1$ only

(c) $0 < k < 1$ only

(d) $k > 2$ only

(e) $1 < k < 2$ only .

26. If f' is continuous, $f(6) = -2$, and $\int_2^3 x f'(10 - x^2)dx = 4$, then $f(1) =$

(a) -10

(b) 6

(c) -12

(d) 1

(e) -3

27. If n is the smallest number of terms that are required to ensure that the sum of the series $\sum_{n=1}^{\infty} \frac{3}{n^4}$ is accurate within 0.004, then $3n+2=$
[You may use $\sqrt[3]{2} \approx 1.26$]

(a) 23

(b) 21

(c) 19

(d) 17

(e) 15

28. The Taylor series for $f(x) = e^{\frac{x}{2}-1}$ centered at $a=2$, is

(a) $\sum_{n=0}^{\infty} \frac{2^{-n}}{n!}(x-2)^n$

(b) $\sum_{n=0}^{\infty} \frac{2^{n}}{(n+1)!}(x-2)^n$

(c) $\sum_{n=0}^{\infty} \frac{2^{-n}}{n!}(x-2)^{n+1}$

(d) $\sum_{n=0}^{\infty} \frac{(x-2)^{-n}}{n!}$

(e) $\sum_{n=0}^{\infty} \frac{2^{-n}}{(n+1)!}(x-2)^{n+1}$