

MATH 101.10 (112)

Quiz 6 (Sects. 4.2-4.4)

Duration: 20mn

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

1.) (4pts) Find the values of  $c$  satisfying the conclusions of the Mean Value Theorem for  $f(x) = \frac{x}{x+2}$  on  $[1, 4]$ .

2.) (3pts) Find the intervals of increase and decrease of the function  $f(x) = \frac{\sin x}{2+\cos x}$  on  $[0, \pi]$ .

3.) (3pts) Find  $\lim_{x \rightarrow 0} \frac{e^{-2x} - 1 + 2x - 2x^2}{x^3}$ .

$$1) f(4) - f(1) = f'(c)(4-1)$$

$$f(1) = \frac{1}{3}, \quad f(4) = \frac{4}{6} = \frac{2}{3}$$

$$f'(x) = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2}$$

So, we have

$$\frac{2}{(c+2)^2} = \frac{1}{9}$$

$$(c+2)^2 = 18$$

$$c+2 = \pm 3\sqrt{2}$$

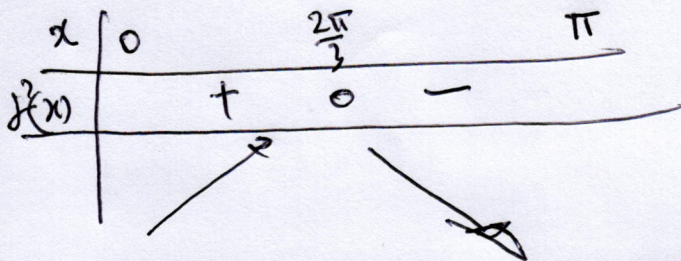
$$c = -2 \pm 3\sqrt{2}$$

but,  $-2 - 3\sqrt{2} \notin [1, 4]$ , so

$$\boxed{c = -2 + 3\sqrt{2}}$$

$$2) f'(x) = \frac{\cos x (2 + \cos x) + \sin^2 x}{(2 + \cos x)^2}$$

$$= \frac{2 \cos x + 1}{(2 + \cos x)^2}$$



$f$  increases on  $[0, \frac{2\pi}{3}]$   
and decreases on  $[\frac{2\pi}{3}, \pi]$

$$3) \lim_{x \rightarrow 0} \frac{e^{-2x} - 1 + 2x - 2x^2}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{-2e^{-2x} + 2 - 4x}{3x^2}$$

$$= \lim_{x \rightarrow 0} \frac{4e^{-2x} - 4}{6x}$$

$$= \lim_{x \rightarrow 0} \frac{-8e^{-2x}}{6}$$

$$= \frac{-8}{6} = \underline{\underline{-\frac{4}{3}}}$$