MATH 101.10 (112)

Quiz 6 (Sects. 4.2-4.4) Duration: 20mn

Name: ID number:

1.)(4pts)Find the values of c satisfying the conclusions of the Mean Value Theorem for $f(x) = \frac{x}{x+2}$ on $[1,4]$.

2.)(3pts)Find the intervals of increase and decrease of the function $f(x) = \frac{\sin x}{2+\cos x}$ on $[0,\pi]$.

3.)(3pts)Find $\lim_{x\to 0} \frac{e^{-2x}-1+2x-2x^2}{x^3}$.

---

1) $f(4) - f(1) = f'(c)(4-1)$

$f(1) = \frac{1}{3}$, $f(4) = \frac{4}{6} = \frac{2}{3}$

$f'(x) = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2}$

So, we have

$\frac{2}{(c+2)^2} = \frac{1}{9}$

$(c+2)^2 = 18$

$c+2 = \pm 3\sqrt{2}$

$c = -2 \pm 3\sqrt{2}$

but, $-2-3\sqrt{2} \notin [1,4]$, so

$\boxed{c = -2+3\sqrt{2}}$

2) $f'(x) = \frac{\cos x(2+\cos x) + \sin^2 x}{(2+\cos x)^2}$

$= \frac{2\cos x + 1}{(2+\cos x)^2}$

x	0		$\frac{2\pi}{3}$		π
$f'(x)$		$+$	0	$-$	

f increases on $[0, \frac{2\pi}{3}]$ and decreases on $[\frac{2\pi}{3}, \pi]$

3) $\lim_{x\to 0} \frac{e^{-2x}-1+2x-2x^2}{x^3}$

$= \lim_{x\to 0} \frac{-2e^{-2x}+2-4x}{3x^2}$

$= \lim_{x\to 0} \frac{4e^{-2x}-4}{6x}$

$= \lim_{x\to 0} \frac{-8e^{-2x}}{6}$

$= -\frac{8}{6} = -\frac{4}{3}$