MATH 101.10 (112)

Quiz 4 (Sects. 3.5-3.9) Duration: 20mn

Name: ID number:

JUSTIFY YOUR ANSWERS.

1.)(4pts)Find $\lim_{x\to\frac{\pi}{4}} \frac{\cot(2x)}{x-\frac{\pi}{4}}$, $\lim_{x\to 0} \frac{\sin x \tan(3x)}{x^3+2x^2}$, $\lim_{x\to 0}(1+2x)^{\frac{3}{x}}$, $\lim_{x\to 0} \frac{5x^2}{2x-2x\cos x+2\sin^2(3x)}$

2.)(3pts) If $f(x) = \cos^{-1}(\ x) + (\sin^{-1} x)^{x+\frac{1}{2}}$, **then find** $f'(\frac{1}{2})$.

3.)(3pts)If $g(x) = 2x^x + 3^x - x^2$, **then find** $g'(1)$.

1a) $\lim\limits_{x\to \pi/4} \frac{\cot(2x)}{x-\pi/4} = \lim\limits_{x\to\pi/4} \frac{f(x)-f(\pi/4)}{x-\pi/4}$,

where $f(x) = \cot(2x)$.

So that, $\lim\limits_{x\to\pi/4} \frac{\cot 2x}{x-\pi/4} = f'(\pi/4)$

$f'(x) = 2\left(-\frac{1}{\sin^2(2x)}\right)$

$f'(\pi/4) = -\frac{2}{\sin^2(\pi/2)} = -2$

$$\boxed{\lim_{x\to\pi/4} \frac{\cot 2x}{x-\pi/4} = -2}$$

$\lim\limits_{x\to 0} \frac{\sin x \tan 3x}{x^3+2x^2} = \lim\limits_{x\to 0} \frac{\frac{\sin x}{x}\frac{\tan 3x}{x}}{x+2}$

But, $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

and $\lim\limits_{x\to 0} \frac{\tan 3x}{x} = \lim\limits_{x\to 0} 3 \frac{\sin 3x}{3x} \frac{1}{\cos 3x}$

$= 3$

thus, $\boxed{\lim\limits_{x\to 0} \frac{\sin x \tan 3x}{x^3+2x^2} = \frac{3}{2}}$

$\lim\limits_{x\to 0} (1+2x)^{\frac{3}{x}} = \lim\limits_{x\to 0} e^{3\frac{\ln(1+2x)}{x}}$

But, $\lim\limits_{x\to 0} \frac{\ln(1+2x)}{x} = \lim \frac{g(x)-g(0)}{x-0}$

$= g'(0)$

$g(x) = \ln(1+2x)$, $g'(x) = \frac{2}{1+2x}$

$g'(0) = 2$

So, $\boxed{\lim\limits_{x\to 0} (1+2x)^{\frac{3}{x}} = e^6}$

$\lim\limits_{x\to 0} \frac{5x^2}{2x-2x\cos x+2\sin^2 3x} = \lim\limits_{x\to 0} \frac{5}{2\left(\frac{1-\cos x}{x}\right)+2\frac{\sin^2 3x}{x^2}}$

But, $\lim\limits_{x\to 0} \frac{1-\cos x}{x} = 0$.

$\lim\limits_{x\to 0} \frac{\sin^2 3x}{x^2} = \lim\limits_{x\to 0} 9\left(\frac{\sin 3x}{3x}\right)^2 = 9$

So $\boxed{\lim\limits_{x\to 0} \frac{5x^2}{2x-2x\cos x+2\sin^2 3x} = \frac{5}{18}}$

2.) $f(x) = \cos^{-1}x + (\sin^{-1}x)^{x+\frac{1}{2}}$

$= \cos^{-1}x + e^{(x+\frac{1}{2})\ln(\sin^{-1}x)}$

$f'(x) = -\frac{1}{\sqrt{1-x^2}} + \left(\ln(\sin^{-1}x) + \frac{(x+\frac{1}{2})}{\sin^{-1}x\sqrt{1-x^2}}\right) e^{(x+\frac{1}{2})\ln(\sin^{-1}x)}$

$= -\frac{1}{\sqrt{1-x^2}} + \left(\ln(\sin^{-1}x) + \frac{x+\frac{1}{2}}{\sin^{-1}x\sqrt{1-x^2}}\right)(\sin^{-1}x)^{x+\frac{1}{2}}$

$f'(\frac{1}{2}) = -\frac{1}{\sqrt{1-\frac{1}{4}}} + \left(\ln \sin^{-1}\frac{1}{2} + \frac{1}{\sin^{-1}\frac{1}{2}\sqrt{1-\frac{1}{4}}}\right)(\sin^{-1}\frac{1}{2})^{1}$

$= -\frac{2}{\sqrt{3}} + \left(\ln\pi/6 + \frac{2}{\frac{\pi}{6}\sqrt{3}}\right)\frac{\pi}{6} = \underline{\pi/6 \ln(\pi/6)}$

3.) $g(x) = 2x^x + 3^x - x^2$

$= 2e^{x\ln x} + e^{x\ln 3} - x^2$

$g'(x) = 2(\ln x + 1)e^{x\ln x} + \ln 3\, e^{x\ln 3} - 2x$

$= 2(1+\ln x)x^x + \ln 3\, 3^x - 2x$

$g'(1) = 2 + 3\ln 3 - 2 \quad = \underline{3\ln 3}$