MATH 101.10 (112)

Quiz 1 (Sects. 1.1-2.3) Duration: 20mn

**Name:** **ID number:**

**1.)(6pts)Evaluate the limit if it exists**

$a.)\lim_{x\to 2}\frac{x^4-16}{x-2},\qquad b.)\lim_{x\to\frac{\pi}{2}}\cos^2 x\sin\left(\frac{\pi}{x-\frac{\pi}{2}}\right),\qquad c.)\lim_{x\to 0^+}\frac{[[x+1]]-x}{|x|-1}.$

**2.)(4pts)Use limits to find all vertical asymptotes of** $y=\frac{x-2}{\sqrt{x^2-1}}$

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1) a) We have

$$\frac{x^4-16}{x-2}=\frac{(x^2-4)(x^2+4)}{x-2}$$

$$=\frac{(x-2)(x+2)(x^2+4)}{x-2}$$

$$=(x+2)(x^2+4),\ \text{if } x\neq 2$$

$$\lim_{x\to 2}\frac{x^4-16}{x-2}=\lim_{x\to 2}\left[(x+2)(x^2+4)\right]$$

$$=32$$

b) We have

$$-1\leq \sin\left(\frac{\pi}{x-\pi/2}\right)\leq 1,\ \text{if } x\neq\frac{\pi}{2}$$

We multiply the inequality by $\cos^2 x$. We find

$$-\cos^2 x\leq \cos^2 x\sin\left(\frac{\pi}{x-\pi/2}\right)\leq \cos^2 x$$

But, $\lim_{x\to\pi/2}\left[-\cos^2 x\right]=0$

and $\lim_{x\to\pi/2}\left[\cos^2 x\right]=0$

By the squeeze theorem, it follows that

$$\lim_{x\to\pi/2}\cos^2 x\sin\left(\frac{\pi}{x-\pi/2}\right)=0$$

c) $\lim_{x\to 0^+}\frac{[[x+1]]-x}{|x|-1}=-1$

2.) We solve $\sqrt{x^2-1}=0$,

$x=-1$ and $x=1$.

Since $x\in(-\infty,-1)\cup(1,+\infty)$

$$\lim_{x\to -1^-}\frac{x-2}{\sqrt{x^2-1}}=-\infty$$

and

$$\lim_{x\to 1^+}\frac{x-2}{\sqrt{x^2-1}}=-\infty$$

So, the vertical asymptotes are $\boxed{x=-1}$ and $\boxed{x=1}$