KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 111

STAT 301: INTRODUCTION TO PROBABILITY THEORY

FINAL EXAM Saturday January 7, 2012

Name: _____ ID #: _____

☺ Important Note:

Show all your work, intermediate steps and final answer.

Question No	Full Marks	Marks Obtained
1	5	
2	5	
3	5	
4	7	
5	8	
6	15	
7	10	
8	8	
9	15	
10	5	
11	7	
Bonus	10	
Total	90+10	

1) A new insurance company offers home owners fire or theft insurance. 24 % of its home owners have fire insurance, 61% have theft insurance and 11% have both.
   a) What percentage of home owners have insurance that the company can offer? *(2pts)*
   b) Of the homes that have theft insurance, what percentage has fire insurance? *(3pts)*

2) A sample of 3 items is selected at random from a box containing 20 items of which 4 are defective. What is the probability that at most one item in the sample is defective? *(5pts)*

3) Monthly sales figures for a particular food industry is normally distributed with mean of 150 (thousand riyals) and standard deviation of 35 (thousand riyals). Find the probability that monthly sales between 100 (thousand riyals) and 200 (thousand riyals). *(5pts)*

4) You and a friend are playing a game in which you throw a fair die 7 times. You bet on 5 and 6, while he bets on the rest. What is the probability that you win the bet? *(7pts)*

5) A box has 10 balls numbered 1 to 10. Two balls are chosen with replacement. Let X denote the larger of the two numbers on the balls selected. Find the probability function of X. *(8pts)*

6) The amount of bread (in hundreds of kilos) that a bakery sells in a day is a random variable with density

$$f(x)=\begin{cases} cx & \text{for } 0 \le x < 3 \\ c(6-x) & \text{for } 3 \le x < 6 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the value of c which makes f a probability density function. *(2pts)*

b) Find the expected amount of bread sold in a day. *(2pts)*

c) Find the CDF of X. *(5pts)*

d) Let A be the event that the number of kilos of bread sold in a day is more than 300 kilos.

B be the event that the number of kilos of bread sold in a day is between 150 and 450 kilos.

e) Are A and B independent? *(6pts)*

7) If X is a standard normal random variable, find the moment generating function of $Y = X^2$, and use it to find the mean of Y. *(10pts)*

8) If the variables X_1 and X_2 are independent, with finite variances. Find $Cov(X_1 + X_2,\ X_2 - X_1)$ *(8pts)*

9) The joint density of X and Y is given by

$$f_{X,Y}(x,y)=\begin{cases} x+y & \textit{for} \quad 0\le x\le 1,\quad 0\le y\le 1 \\ 0 & \textit{otherwise} \end{cases}$$

Find the correlation coefficient $\rho(X,Y)$ *(15pts)*

10) An insurance company has 10000 automobile policy holders. The expected yearly claim per policy holder is \$240 with a standard deviation of \$800. Approximate the probability that the total yearly claim exceeds \$2.7 million. *(5pts)*

11) Let X be a Poisson random variable with parameter λ. Use Chebyshev's inequality to show that $P\left(X \leq \frac{\lambda}{2}\right) \leq \frac{\lambda}{4}$. *(7pts)*

Bonus: ***(10pts)***

Define the probability generating function of a non-negative discrete random variable X as $G_X(t) = E\left(t^X\right), \quad |t| \le 1$.

a) If X is a binomial random variable, find its probability generating function.
b) Use it to find the probability that $X = 1$.