Department of Mathematics and Statistics
Semester 111

STAT301 First Major Exam Monday November 14, 2011

Name: ______________________ ID #: ____________________

1) A family has 4 children. Let B denote a boy and G denote a girl. Write down the following events: *(9 pts.)*
   i) A: boys and girls alternate

   ii) B: the first and the fourth child are boys

   iii) C: as many boys as girls

   iv) D: three successive children of the same sex

2) In how many ways can a lady having 10 dresses, 5 pairs of shoes, and 2 hats be dressed? *(3 pts.)*

3) Given that P(A) = 1/3, P(B) = ¼, P(AB) = 1/6. Find the following probabilities: *(7 pts.)*

   i) $P(A^c)$

   ii) $P(A^c \cup B)$

   iii) $P(A^c \cup B^c)$

   iv) $P(A^c B^c)$

4) A ball is selected at random from a box containing n balls labeled 1, 2, …, n. What is the probability that its label is divisible by 3 or 4? *(6 pts.)*

5) A die is thrown as long as necessary for a 1 or a 6 to turn up. Given that no 1 turned up at the first two throws, what is the probability that at least three throws will be necessary? *(7 pts.)*

6) Urn A contains 5 black balls and 6 white balls, and urn B contains 8 black balls and 4 white balls. Two balls are transferred from B to A and then a ball is drawn from A. What is the probability that this ball is white? *(8 pts.)*

7) Ten percent of a certain population suffer from a serious disease. A person suspected of the disease is given two independent tests. Each test makes a correct diagnosis 90% of the time. Find the probability that the person really has the disease given that both tests are positive. *(10 pts.)*

Bonus Question: *(5 pts.)*

Show that for every integer n ≥ 2,

$$1-\binom{n}{1}+\binom{n}{2}+\cdots+(-1)^n\binom{n}{n}=0$$