King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 590 , Term: 111
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HOMEWORK – 6
Due Wednesday 4 January, 2012

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1) Consider the problem Ax=b with

$$A = \begin{bmatrix} \varepsilon & 1 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1+\varepsilon \\ 2 \end{bmatrix}$$

where ε is a small parameter (to be determined below)

(a) Is A ill or well conditioned? (eg. Find $cond_1(A)$)

(b) Find the LU decomposition of A (by hand)
   i. without partial pivoting
   ii. with partial pivoting

(c) For $\varepsilon = 10^{-2k}$ where k=1,…,10, use matlab to calculate the solution to Ax=b without partial pivoting (use your result above), and **make a plot** of the relative error $r = \|x - x_{true}\| / \|x_{true}\|$ where $x_{true} = [1 \quad 1]^T$ is the exact solution. What is the largest value of k (or smallest value of ε) such that the solution is good to 6 digit of decimal precision؟

(d) Repeat the problem but use matlab backslash (i.e. x=A\b) wich implements partial pivoting. What is the maximum relative error for this problem with pivoting؟

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2) Consider the linear system, Ax=b, where A is the following matrix،

$$A = \begin{pmatrix} -5 & 2 & -1 \\ 1 & 0 & 3 \\ 3 & 1 & 6 \end{pmatrix}$$

(a) Using partial pivoting technique, determine the P, L, U decomposition of the matrix A, such that (Show **EACH STEP** in the decomposition)

(b) Use the P, L, U decomposition found in (a) to find the solution to $Ax = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ (Show **ALL** relevant steps)

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3) Download the files: (rowindex.txt; columnindex.txt; matrixa.txt and b.txt). Use the command ***load*** to load the matrix A in sparse form and the vector b.

(a) Solve the linear system Ax=b by using GMRES (find # of iterations and CPU-time).

(b) Find a good preconditioner M for the system which reduces the iteration number and the CPU-time. (find # of iterations and CPU-time).

(c) Compute *mark*, where

$$A = \frac{\#\text{iteration from part (b)}}{\#\text{iteration from part (a)}}$$

$$B = \frac{\text{CPU - time from part (b)}}{\text{CPU - time from part (a)}}$$

$$C = \frac{\|b - Ax_n\|}{\|b\|}$$

$$mark = (1 - A) * 33 + (1 - B) * 33 + (1 - C) * 33$$