King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 590 , Term: 111
Instructor: Dr. Faisal Fairag; Dr. Hattan Tawfiq

HOMEWORK – 4
Due Monday 24 October, 2011

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1) (a) Let $A \in R^{m\times n}$. Use SVD to find the Schur decomposition of the symmetric matrices $A^T A,\ AA^T,\ \begin{bmatrix} 0 & A^T \\ A & 0 \end{bmatrix}$.

(b) Let $A \in C^{n\times n}$ and $\det(\lambda I - A) = c_0 + c_1\lambda + \ldots + c_{n-1}\lambda^{n-1} + \lambda_n$ .For given $x \in C^n$ the matrix $K = \left[x, Ax, \ldots, A^{n-1}x\right] \in C^{n\times n}$ is called the Krylov matrix of A at x.
Suppose that $Q^T AQ = H$ is a transformation of A to upper Hessenberg form with some orthogonal matrix Q. Let q_1 be the first column of Q and from the Krylov matrix of A at $x = q_1$. Show that $K = Q(e_1, He_1, \cdots, H^{n-1}e_1)$.

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2) The boundary value problem

$$\frac{d}{dx}\left[a(x)\frac{dy}{dx}\right] + \lambda y = 0, \quad y(-1) = y(1) = 0$$

is approximated by the eigenvalue problem

$$-\frac{1}{h^2}\left[a(x_k + \frac{h}{2})(z_{k+1} - z_k) - a(x_k - \frac{h}{2})(z_k - z_{k-1})\right] = \lambda_{zk}, \quad h = 1, \cdots, n$$

where $x_k = -1 + kh,\ h = 2/(n+1)$, and $z_0 = z_{n+1} = 0$. Let $a(x) = 1 + x^2$. Use QR algorithm with shift to compute the smallest eigenvalue for n=10,20.

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3) Let $A \in R^{m\times n}$, m > n.

a) write a MATLAB function [Q,L] = ql(A) that compute the matrix Q and L (lower triangular) which you developed in problem 2 HW3.
b) Apply the following Algorithm (which uses function ql in part a)

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function [d]=ql_algoritm(A)
n=size(A);
for k=1:maxiter
   [Q,L]=ql(A);
   A=L*Q;
end
```

to the matrix $B = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$.

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4) Compute the eigenvalues of the matrix A by using three different methods described below then fill the table.

METHOD-1: MATLAB command eig
METHOD-2: QR Algorithm (basic version –without shift)
METHOD-3: QR Algorithm with shift $\mu_k = h_{nn}^{(k)}$ (first find Hessenberg H)
METHOD-4: QR Algorithm with Wilkinson shift (first find Hessenberg H)

	Number of iteration in QR Algorithm	CPU-time (use tic-toc)
METHOD-1		
METHOD-2		
METHOD-3		
METHOD-4		

$$A = \begin{bmatrix} B & -I & 0 & 0 \\ -I & B & -I & 0 \\ 0 & -I & B & -I \\ 0 & 0 & -I & B \end{bmatrix}$$ where B is defined in problem 3.