King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 590 , Term: 111
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HOMEWORK – 2
Due Monday 3 October, 2011

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1) Use the singular value decomposition to show that for any matrix $A \in R^{n\times m}, n \geq m,$ there exist matrices $Q \in R^{n\times m}$ and $B \in R^{m\times m}$ such that $A = QB$ where Q has orthnormal columns, (i.e., $Q^T Q = I_m$) and B is symmetric positive semi-definite, (i.e., $B^T = B,$ and $x^T Bx \geq 0$ for all $x \in R^m$). This decomposition is usually called the polar decomposition of A because it is analogous to the polar representation $z = |z|e^{i\phi}$ of a complex number.

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2) for the matrix $A \in R^{m\times n}$ let $\sigma_1 \geq \sigma_2 \geq .... \geq \sigma_p, p = \min\{m,n\},$ denote the singular values.

(a) If $A^T A = I_m$ what are the singular values of A?

(b) Show that $\|A\|_F^2 = \sigma_1^2 + .. + \sigma_p^2.$

(c) Use (b) to show that $\|A\|_F^2 \leq rank(A)\|A\|_2^2.$

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3) Example 3.6 (p22) shows that if E is an outer product $E = uv^*$, then $\|E\|_2 = \|u\|_2\|v\|_2$. Is the same true for the Frobenius norm, i.e., $\|E\|_F = \|u\|_F\|v\|_F$? Prove it or give a counterexample.

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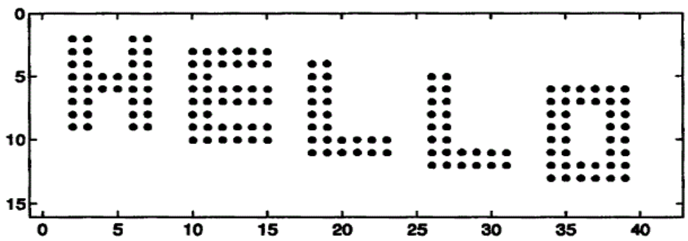
4) Solve exercise 9.3 page 68. (upload your file through WebCT under assignments).

9.3. (a) Write a MATLAB program that sets up a 15×40 matrix with entries 0 everywhere except for the values 1 in the positions indicated in the picture below. The upper-leftmost 1 is in position $(2, 2)$, and the lower-rightmost 1 is in position $(13, 39)$. This picture was produced with the command `spy(A)`.



(b) Call `svd` to compute the singular values of A, and print the results. Plot these numbers using both `plot` and `semilogy`. What is the mathematically exact rank of A? How does this show up in the computed singular values?

(c) For each i from 1 to rank(A), construct the rank-i matrix B that is the best approximation to A in the 2-norm. Use the command `pcolor(B)` with `colormap(gray)` to create images of these various approximations.