King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 590 , Term: 111
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HOMEWORK – 1
Due Monday 26-9-2011

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1) Show that if $S^T = -S$ then $I - S$ is nonsingular and the matrix $(I-S)^{-1}(I+S)$ is orthogonal. This is known as the *Cayley transform* of S.

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2) Show that an upper triangular orthogonal matrix is diagonal. (An upper triangular matrix is a special kind of square matrix where all the entries below the main diagonal are zero.)

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3) A matrix $A \in R^{n\times n}$ is called elementary if it has the form

$$A = I_n - \alpha uv^T, \qquad u, v \in R^n, \alpha \in R$$

(a) Determine γ such that $(I_n - \beta uv^T)(I_n - \alpha uv^T) = (I_n - \gamma uv^T)$

(b) Show that $(I_n - \alpha uv^T)^{-1} = (I_n - \beta uv^T)$ if and only if $\alpha uv^T \neq 1$ and determine β.

(c) Show that $U = I_n - \alpha uv^T$ with $u \neq 0$ and $\alpha = \dfrac{2}{u^T u}$ is symmetric $(U^T = U)$, orthogonal $(U^T U = I_n)$ and $(U^2 = I_n)$ involuntary.

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4) In matrix computations, storage and number of operations is of considerable importance. A sparse matrix is one with a large percentage of zero elements. When dealing with large, sparse matrices, it is desirable to take advantage of the sparsity by storing and operating only on the nonzeros. In programming, just the nonzero entries of the matrix together with row and column indices are stored. Let A be an nxn matrix and given three vectors $a, acol, arow$ of length nnz. Then $A(acol(k), arow(k)) = a(k),$ for $k = 1,2,\cdots,nnz$. The following program computes the matrix-vector product $b = Ax$:

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for i=1:n
   for j=1:n
       b(i)=b(i)+A(i,j)*x(j);
   end
end
```

In this program the matrix A is given in a full form not a sparse form.

(a) Write a function m-file

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function [b] = matrix_vector_product(acol,arow,a,x)
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which read the matrix A (stored in acol, arrow, a) and the vector b then perform matrix-vector product and store the product in the vector b. The function should be applicable for arbitrary dimensioned vectors.

(b) Test you function with the following inputs:
$acol$ = [1 2 3 4 5 11 110 305 306 505 506 997 998 999 1000]
$arow$ = [1 2 3 4 5 890 790 300 301 500 501 997 998 999 1000]
a = [2 2 2 2 2 4 4 3 3 1 1 1 1 1 1]
x = [1 2 3 4 ….. 997 998 999 1000]

(c) compute $\|b\|_2$ in part (b) (you can use the built-in command <u>norm</u>)

(d) Use MATLAB built-in command sparse to store the matrix A and perform the matrix-vector product and verify your result in part (b).