King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 590 Exam II
Semester I, (111)
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Name:	
ID:	

Q		Points
1		30
2		30
3		40
4		40
5		40
6		40
Total		220

(1) Suppose D is a diagonal 805x805 matrix with diagonal entries 1.00, 1.01, . . . , 8.99, 9.00. and -12, -13, -20, -24. V is an orthogonal matrix. If $A = V^T DV$. How many steps of the MINRES iterations must you take to be sure of reducing the initial residual $\|r_0\|_A$ by a factor 10^6 ?

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(2) Suppose A is a dense symmetric positive definite 1000x1000 matrix with $\kappa(A) = 100$. Estimate roughly how many flops are required to solve $Ax = b$ to ten-digit accuracy by Conjugate Gradient method.

(3) Suppose H is symmetric tridiagonal 100x100 matrix with eigenvalue $\lambda = 2.123456789$. After k steps of QR-algorithm with Wilkinson shift, we have $h_{100,99} = 0.1153, h_{100,100} = 2.153$. Which one of the following could be the value $h_{100,99}$ at k+1 step.

a) 0.112 b) 0.013 c) 0.004 d)0.0013

Reason for your choice: ……..
…….
…….
……

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(4) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix}$

Find orthogonal matrices U and V so that $U^T AV = B =$ Bidiagonal matrix. (show all your work)
[Note: In phase one of the SVD computing, we first convert the matrix A into bidiagonal matrix]

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(5) consider the following linear system $\begin{bmatrix} 6 & 2 & 7 \\ 2 & 3 & 2 \\ 7 & 2 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Which one of the following statement is true:

(a) Jacobi method converges faster that GS
(b) GS method converges faster than the Jacobi
(c) Both Jacobi and GS methods will diverge
(d) Both Jacobi and GS methods converge

Reason for your choice: ……..
…….
…….
……

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(6) True of False (circle T or F)

I) Let A be an mxm symmetric positive definite matrix. If exact arithmetic is used then conjugate gradient iterations converges in at most m steps……..…….….. (T F)

II)Let A and B be orthogonal matrices. Then A is orthogonaly similar to B ….. (T F)

III)If all eigenvalues of T are less than one in modulus, then the iteration $x_{k+1} = Tx_k$ converges to the zero vector as $k \to \infty$ ……..……..….. (T F)

IV)Both MINRES and Conjugate Gradient methods are three-term recurrence based methods.
……..……..….. (T F)

V)IF $q_1, q_2, \cdots, q_n$ are the orthonormal vectors come from Arnoldi iterations. Let $Q_n = [q_1, q_2, \cdots, q_n]$ and $K_n = [b, Ab, \cdots, A^{n-1}b]$ be two matrices with n-columns Then $Q_n^T K_n$ is an upper square matrix……..……..……..….. (T F)