King Fahd Univ. of Petroleum and Minerals
Faculty of Sciences
Department of Mathematics and Statistics

FINAL EXAM
MATH. 533-111

Prob. 1

(a) Find all Laurent series of $f(z) = \frac{z+5}{z^2-2z-3}$ centered at the origin and indicate the annulus of convergence of each series.

(b) Find the expansion of $g(z) = \frac{z^3-3z^2+3}{(z-1)(z-3)}$ in powers of z and z^{-1} in the regions: (i) $|z| < 1$, (ii) $1 < |z| < 3$ and (iii) $|z| > 3$.

Prob. 2

Find the image of the upper half-plane by $w = \int_0^z \frac{dt}{\sqrt{1-t^2}}$.

Prob. 3

Determine the function that maps the slit of height "s" onto the upper half of the $z-$plane

Prob. 4

Determine the function that maps the exterior of the isocele triangle located in the upper half of the w-plane onto the upper half of the $z-$plane.

Prob. 5

Find a necessary and sufficient condition for a bilinear transformation to map the upper half-plane $\mathbf{I}z > 0$ onto the unit disk $|w| < 1$.

(Determine the explicit form of the transformation).

Prob. 6

Find a necessary and sufficient condition for a bilinear transformation to map the disk $|z| < 1$ onto the unit disk $|w| < 1$.

(Determine the explicit form of the transformation).

Prob. 7

Find a conformal mapping f of the semi-disk $|z| < 1$, $\mathbf{I}z > 0$ onto the upper half-plane.

Prob. 8

Find, using the Schwarz-Christoffel formula, the mapping that maps the channel in the figure below onto the upper half-plane.