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MAJOR No. 2

MATH. 533-111

Prob. 1

Evaluate $\int_\gamma \frac{z}{\bar{z}} dz$ where γ is the boundary of the upper half of the annulus $\{z : 1 < |z| < 2\}$ with positive orientation.

Prob. 2

Prove that for any positive integers m, n and a, b such that $n(\gamma, a) = n(\gamma, b)$ we have $\int_\gamma \frac{dz}{(z-a)^m (z-b)^n} = 0$.

Prob. 3

Suppose P_n is a polynomial of degree at most equal to n and $a \in C(0, r)$. Prove that $\int_{C(0,r)} \frac{P_n(z)dz}{z^{n+1}(z-a)} = 0$.

Prob. 4

If $|a| < r < |b|$, show that $\int_{C(0,r)} \frac{dz}{(z-a)(z-b)} = \frac{2\pi i}{a-b}$.

Prob. 5

Evaluate $\int_0^{2\pi} \log \left| re^{i\theta} - a \right| d\theta$, $r < |a|$.

Prob. 6

Evaluate $\int_{C(0,r)} \frac{dz}{(z-a)^m (z-b)}$, $|a| < r < |b|$.

Prob. 7

Suppose f is a non-constant analytic function in $C(a; r)$. Show that there exists a positive integer m such that $f^{(m)}(a) \neq 0$.

Prob. 8

Prove that $\mathbf{R}s \left\{ \left[(1 + z^2) \cosh \frac{\pi z}{2} \right]^{-1} ; i \right\} = \frac{1}{2\pi i}$.

Prob. 9

Prove that $\int_0^{2\pi} \frac{\cos^2 3\theta d\theta}{1+a^2-2a\cos 2\theta} = \pi \frac{1-a+a^2}{1-a}$, $0 < |a| < 1$.

Prob. 10

Show that

(i) $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx = \frac{5\pi}{12}$

(ii) $\int_{-\infty}^{\infty} \frac{x^6 dx}{(x^4+a^4)^2} = \frac{3\sqrt{2}\pi}{16a}$, $a > 0$.