KING FAHD UNIVERSITY OF PETROLEUM AND MINERAL

Department of Mathematical Sciences

Final Exam MATH - 521 Sem 111

Student #:_____________ Name:____________________

Show All Your Work. No Credits for Answers Not Supported by Work.

Q1) (10 Points) Give complete definition of each of the following:

a. Path connected space

b. Lindelof space

c. Countably compact

d. Heine –Borel Theorem

e. Completely normal space

Q2) (40 Points) Give an example of each of the following and describe it in few lines.

(a) A topological space which is not metrizable.

(b) A non-empty closed set with no limit points.

(c) A continuous function which is not uniformly continuous.

(d) A first countable space which is not second countable.

(e) A Housdorff space which is not regular.

(f) A regular space which is not T_1.

(g) The intersection of two compact sets which is not compact.

(h) A closed bounded subset of a metric space which is not compact.

(i) Separable space which is not second countable.

(j) Compact space which is not locally compact.

Q3) (70 Points) Prove only 7 problems.

1. If $(X_i, \Im_i)$ is Housdorff space for each $i = 1, 2, 3, \cdots, n$, prove that $\prod_{i=1}^{n} X_i$ is Housdorff.
2. Let $(X, \Im)$ be Housdorff space and let *F* be a compact subset of *X*. If a point $x \in X$ is not in *F*, then there exist disjoint open sets *U* and *V* such that $x \in U$ and $F \subseteq V$.
3. Every compact subspace of a Housdorff space is closed.
4. A 1-1 continuous mapping from a compact space into a Housdorff space is homeomorphism.
5. Every separable metric space is second countable.
6. In a first countable space *X*, if a point *x* is a limit point of a subset *A* of *X*, then there exists a sequence in $A - \{x\}$ which converges to x.
7. If a space contains a connected dense subset then it must be connected.
8. Let X be T_1 countably compact space. Prove that each infinite subset of X has at least one limit point in *X*.
9. Every path connected space is connected.