King Fahd University of Petroleum and Minerals
Departement of Mathematics and Statistics
MATH 430 (Semester 111)
Final Exam

January 14, 2012

Exercise 1

For the function

$$\frac{z}{(z+1)(z-2)}$$

find the Laurent series expansion in the following domains

(a) $1 < |z| < 2$;

(b) $|z| > 2$.

Exercise 2

Let $f(z)$ have an isolated singularity at z_0 and suppose that $f(z)$ is bounded in some punctured neighborhood of z_0.
Prove directly from the integral formula for the Laurent coefficients that $a_{-j} = 0$ for all $j = 1, 2, \ldots$; that is, $f(z)$ must have a removable singularity at z_0.

Exercise 3

(a) Find all the functions f that are analytic everywhere in the extended complex plane.

(b) Find all the function f that are analytic everywhere in the extended complex plane except for a pole at infinity.

Exercise 4

Let ω be a complex number such that $Im(\omega) > 0$. Compute

$$\text{p.v.} \int_{-\infty}^{\infty} \frac{\cos x}{x - \omega} dx$$

Exercise 5

Compute

$$\int_0^{\infty} \frac{\sin(2x)}{x(x^2+1)^2}\, dx$$

Exercise 6

Find the number of roots of the equation $6z^4 + z^3 - 2z^2 + z - 1 = 0$ in the disk $|z| < 1$.