King Fahd University of Petroleum and Minerals
Departement of Mathematics and Statistics
MATH 430 (Semester 111)
Major II

December 8, 2011 Allowed Time: 2 hours

Exercise 1

1. Show that $\sin(x+iy) = \sin x \cosh y + i \cos x \sinh y$.
2. Find all roots of the equation $\sin z = \cosh 4$.

Exercise 2

1. Find all values of $(1+i)^i$.
2. Find the principal value of $(1-i)^{4i}$.

Exercise 3

Define a branch of $(z^3-1)^{\frac{1}{3}}$ that is analytic in the exterior of the unit circle, $|z|>1$.

Exercise 4

1. Show that $\tan^{-1} z$ is a multiple valued function defined by
$$\tan^{-1} z = \frac{i}{2} \log \frac{1-iz}{1+iz}$$
2. Precise the branch cut for $\mathrm{Tan}^{-1}\mathrm{z}$, the *principal* values of $\tan^{-1} z$, and the range of $\mathrm{Tan}^{-1}x$, $x \in \mathbb{R}$.

Exercise 5

Evaluate
$$\int_\Gamma \frac{2z^2 - z + 1}{(z-1)^2(z+1)} dz$$
where Γ is the circle $|z| = 2$ traversed once in the counterclockwize direction.

Exercise 6

Let $\mathcal{C}$ be any simple closed contour, described in the positive sense in the z-plane and we write
$$g(z) = \int_{\mathcal{C}} \frac{w^3 + 2w}{(w-z)^3} dw$$
1. Show that $g(z) = 6\pi i z$ when z is inside $\mathcal{C}$.
2. What is the value of $g(z)$ when z is outside $\mathcal{C}$?