King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 321 Final Exam

Term 111

Full name:……………………………………………….
ID Number:………………………………………….

Question Number	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Total
Total	14	6	6	8	8	13	10	10	10	5	6	14	15	125
Your Mark														

1- let $f(x) = x^3 - 3x + 2$

(a) Find the zeros of f .

(b) Use the Newton Raphson Method to find P_1, P_2 and P_3 starting with $P_0 = 1.2$.

(c) Use Accelerated Newton Raphson method to find P_1, P_2 and P_3 starting with $P_0 = 1.2$.

(d) What is your conclusion?

2- Consider the system $Ax = b$ where $A = \begin{bmatrix} 3 & a & 2 \\ -1 & 2 & b \\ 9 & 2 & 12 \end{bmatrix}$ and a and b are non negative integers.

Find a and b such that Jacobi or Gauss-Seidel iterations will converge starting with any initial iteration.

3- Let $f(x)$ be a polynomial of degree $\leq N$. Let $P_N(x)$ be the Lagrange polynomial of degree $\leq N$ based on the $N+1$ nodes $x_0, x_1, \ldots, x_N$. Show that $f(x) = P_N(x)$ for all x .

4- Find a bound on the maximum error $\left(|E_2(x)|\right)$ on the interval $[0, \pi]$, when the Newton interpolating Polynomial $P_2(x)$ is used to approximate $f(x) = \cos(\pi x)$ at the centers $x_0 = 0,\ x_1 = \frac{\pi}{2}$ and $x_2 = \pi$.

5- a) Carry out the change of variables and constants $X = xy$, $Y = y$, $C = -\frac{1}{A}$ and $D = -\frac{B}{A}$ to derive the linearized form $Y = AX + B$ for the function $y = \frac{D}{x + C}$.

b) Carry out the change of variables and constants $X = x$, $Y = \ln\left(\frac{y}{x}\right)$, $C = e^B$ and $D = -A$ to derive the linearized form $Y = AX + B$ for the function $y = Cxe^{-Dx}$.

6- Let $f(x) = \cos x$

(a) Write the Formulas for

I. Centered Formula of order $O(h^2)$.

II. Centered Formula of order $O(h^4)$.

(b) Use I and II with step size $h = 0.01$ to calculate approximations for $f'(0.8)$.

(c) Find the absolute errors of using I and II.

(d) Use Richardson's Extrapolation to find the approximate formulas of order $O(h^6)$ and $O(h^8)$ in terms of the previous formulas.

7- Derive the formula $f''(x_0) \approx \frac{2f_0 - 5f_1 + 4f_2 - f_3}{h^2}$.

Hint: Start with the Lagrange Polynomial for $f(t)$ based on the four points x_0, x_1, x_2 and x_3.

8- Determine the degree of precision of Simpson's $\frac{3}{8}$ rule.

Hint: You may apply Simpson's $\frac{3}{8}$ rule over the interval $[0,3]$.

9- a) Approximate $\int_0^4 x^2 e^{-x} dx$ using the composite trapezoidal rule with $M = 10$.

b) Approximate the same integral using the composite Simpson rule with $M = 5$.

10- Determine the smallest integer k for which $\int_0^2 10x^9 dx = R(k,k)$.

11- Derive the Recursive Boole Rule using Romberg Integration starting with $R(J,0) = T(J)$ where $T(J) = T(f,h)$ is the sequential Trapezoidal Rule.

12- Consider the initial Value Problem (IVP):

$$\frac{dy}{dt} = -ty, \qquad t \in [0,1] \text{ with } \qquad y(0) = 1$$

a) Show that IVP has a unique solution.

b) Find the exact solution of the given IVP.

c) Use Euler method to solve the given IVP using step size of length $h = \frac{1}{3}$.

d) Compute the absolute error between the exact solution and the Euler solution with the step size h.

13- Use the Runge-Kutta Method of order 4 to solve the IVP

$$\frac{dy}{dt} = \frac{t-y}{2}, \qquad t \in [0,3] \text{ with } y(0) = 1$$

using step size of length $h = 1$.