King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 321 Second Exam

Term 111

Full name:……………………………………………….
ID Number:…………………………………………….

Question Number	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
Total	10	10	10	10	10	10	10	70
Your Mark								

1- (10 points) Consider the system

$$4x - y = 15$$
$$x + 5y = 9$$

(a) Start with $P_0 = 0$ and use Jacobi iteration to find P_k for $k = 1,\ 2,\ 3$. Will Jacobi iteration converges to the solution?

(b) Start with $P_0 = 0$ and use Gauss-Seidel iteration to find P_k for $k = 1,\ 2,\ 3$. Will Gauss-Seidel iteration converges to the solution?

(c) Discuss the convergence in a) and b) above. WHY?

2- (10 points) Let $f(x) = (2+x)^{\frac{1}{2}}$.

(a) Find the Taylor Polynomial $P_3(x)$ expanded about $x_0 = 2$.

(b) Use $P_3(x)$ to find an approximation to $3^{\frac{1}{2}}$.

(c) Find the maximum value of $\left|f^{(4)}(c)\right|$ on the interval $1 \le c \le 3$ and find a bound for $\left|E_3(x)\right|$.

3- (10 points) Let $f(x) = x + \frac{2}{x}$.

(a) Use cubic Lagrange interpolation based on the nodes $x_0 = 0.5$, $x_1 = 1$, $x_2 = 2$ and $x_3 = 2.5$ to approximate $f(1.5)$.

(b) Determine the error bounds for the cubic Lagrange interpolation in (a).

4- (10 points) Given the function $f(x) = x^{\frac{1}{2}}$ and the nodes $\{4, 5, 6, 7\}$

(a) Compute the divided-difference table for the function f at these nodes.

(b) Write down the Newton Polynomials $P_2(x)$ and $P_4(x)$.

(c) Evaluate the Newton Polynomials in part (b) at $x = 4.5$.

(d) Compute the absolute and relative errors for the values obtained in part (c).

5- (10 points) For the date

x_k	y_k	$f(x_k)$
-2	1	1.2
-1	2	1.9
0	3	2.6
1	3	3.3
2	4	4.0

Find the least-squares line $y = Ax + B$ for the data and calculate the root-mean square error of f

6- (10 points) For the date, use linearization to find the least-squares curve:

x_k	y_k
1	0.6
2	1.9
3	4.3
4	7.6
5	12.6

a) $f(x) = Ce^{Ax}$, by using the change of variables $X = x$, $Y = \ln(y)$ and $C = e^{B}$

b) $f(x) = Cx^{A}$, by using the change of variables $X = \ln(x)$, $Y = \ln(y)$ and $C = e^{B}$

c) Use the root-mean square error of f to determine which curve gives the best fit.

7- (10 points) Find the natural cubic spline that passes through the points $(-3,2)$, $(-2,0)$, $(1,3)$, and $(4,1)$ with the free boundary conditions $S''(-3)=0$ and $S''(4)=0$.