King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 321 First Exam

Term 111

Full name:……
ID Number:……..

Question Number	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Total
Total	4	5	5	11	6	3	11	8	7	60
Your Mark										

1- (4 points) Let $x = 1000000$ and $y = 999996$. Then

a) Find the error E_X.

b) Find the relative error R_x.

c) Determine the number of significant digits in the approximation.

2- (5 points) Which formulas shall we use to compute the two roots of the equation $x^2 + bx - 10^{-12} = 0$ more accurately where b is a number greater than 100?

3- (5 points) If $a = 0.9$ and $b = 1.0$, how many steps of the bisection method are needed to determine the root with an error of at most $\frac{1}{2} \times 10^{-8}$?

4- (11 points) Consider the equation $x^3 = 2x + 2$.

a) Apply two steps of the method of False Position with initial bracket $[1,2]$ to solve the equation.

b) Apply two steps of the Secant Method to solve the equation with initial guesses $x_0 = 1$ and $x_1 = 2$.

5- (6 points) If Newton's method is used on $f(x) = \frac{1}{x}$ starting with $x_0 = 1$, then what will be x_{50}?

6- (3 points) Complete the following table about the methods for finding zeros of $f(x)$ by specifying the order of convergence R:

Method	Special consideration	$R =$
Newton	Simple Root	
Newton	Multiple Root	
Secant	Simple Root	

7- (11 points) Solve the following system $\begin{bmatrix} 1.133 & 5.281 \\ 24.14 & -1.210 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6.414 \\ 22.93 \end{bmatrix}$ using an 4-digit rounding arithmetic.

a) By using Naïve Gaussian Elimination.

b) By using Gaussian Elimination with partial pivoting.

c) Compare the obtained results from a) and b).

d) Is the given system ill conditioned?

8- (8 points) Consider the matrix $\begin{bmatrix} 25 & 0 & 0 & 0 & 1 \\ 0 & 27 & 4 & 3 & 2 \\ 0 & 54 & 58 & 0 & 0 \\ 0 & 108 & 116 & 6 & 0 \\ 100 & 0 & 0 & 0 & 24 \end{bmatrix}$

a) Determine the LU Factorization.

b) Use a) to solve the system $Ax = b$ with $b = \begin{bmatrix} 26 \\ 36 \\ 112 \\ 230 \\ 124 \end{bmatrix}$.

9- (7 points) Investigate the nature of the fixed point iteration when $g(x) = -4 + 4x - \frac{1}{2}x^2$.

a) Find the fixed points.

b) Use the starting point $p_0 = 1.9$ and compute p_1, p_2 and p_3.

c) Use the starting point $p_0 = 3.8$ and compute p_1, p_2 and p_3.

d) Do we have convergence in a) or b)? Why?