King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 302 Final Exam

Semester (111) January 09, 2012 Time: 07:30 - 10:30 am

Name: ....

I.D: .... Section: ....

Problem	Points
1	____ / 7
2	____ / 9
3	____ / 7
4	____ / 9
5	____ / 7
6	____ / 9
7	____ / 11
8	____ / 11
Total	____ / 70

Problem 1. Let Σ be the portion of the cone $z = \sqrt{x^2 + y^2}$ in the first octant for $1 \leq z \leq 3$. Evaluate the surface integral

$$\iint_{\Sigma} xz^2 \, \mathrm{d}S.$$

Problem 2. Given the surface Σ which consists of the cylinder

$$\Sigma_1 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1,\ 0 \leq z \leq R\}$$

and the disk

$$\Sigma_2 = \{(x, y, z) \in \mathbb{R}^3 : z = R,\ x^2 + y^2 \leq 1, \}.$$

The surface Σ is bounded by the circle C in the xy-plane oriented counterclockwise from above.

Let $F(x, y, z) = y^3\mathbf{i} - x^3\mathbf{j} + \sin(z^2)\mathbf{k}$.

Use Stokes' Theorem to compute the line integral $\oint_{\mathcal{C}} F.\mathrm{d}r$.

Problem 3. Let $F(x, y, z) = 2x\mathbf{i} + z^2\mathbf{j} - y^3x\mathbf{k}$ and M be the volume enclosed between the spheres $S_1 : x^2 + y^2 + z^2 = 4$ and $S_2 : x^2 + y^2 + z^2 = 9$.

Find the outward flux

$$q = \int\int_{S_1 \cup S_2} F.\mathbf{n}\, \mathrm{d}S.$$

Problem 4. Let $f(z) = z|z|^2$. We denote by $x = \mathrm{Re}(z)$, $y = \mathrm{Im}(z)$, $u(x, y) = \mathrm{Re}(f(z))$ and $v(x, y) = \mathrm{Im}(f(z))$.

(1) Find all (x, y) at which u, v satisfy Cauchy-Riemann Equations.

(2) Is $f(z)$ differentiable at $z_0 = 0$? why?

Problem 5. Let $\mathcal{C}$ be the positively oriented circle given by $|z - i| = 1$. Compute the integral

$$\oint_{\mathcal{C}} \frac{|z|^2}{z - i} \,\mathrm{d}z.$$

Problem 6. Use Cauchy Integral Formula to evaluate the integral

$$\oint_{C} \frac{e^{2iz}}{(z+i)^4}\,\mathrm{d}z,$$

where C is any positively oriented simple closed contour enclosing $-i$.

Problem 7.

(1) Find the Laurent series of the function $f(z) = \dfrac{1}{4z - z^2}$ in the annular domain D given by: $0 < |z| < 4$ (punctured disk).

(2) Use (1) to evaluate the contour integral

$$I = \oint_{\mathcal{C}} f(z)\, \mathrm{d}z,$$

where $\mathcal{C}$ is the positively oriented circle $|z| = 3$.

Problem 8. Let $f(z) = \dfrac{1}{z(e^z - 1)}$.

(1) Find all the poles of $f(z)$ and their orders.

(2) Let $\mathcal{C}$ be the positively oriented circle $|z| = 7$. Evaluate the contour integral

$$\oint_{\mathcal{C}} f(z)\,\mathrm{d}z.$$