King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 302 Exam I

Semester (111) October 13, 2011 Time: 1:00 - 2:30 pm

Name: .....

I.D: ..... Section: .....

Problem	Points
1	_____ 10
2	_____ 10
3	_____ 15
4	_____ 15
Total	_____ 50

Exercise 1. Let $S = \{(a, 4a, b, a + b) \mid a, b \in \mathbb{R}\}$.

(i) Show that S is a subspace of $\mathbb{R}^4$.

(ii) Find a basis of S and evaluate $\dim(S)$.

Exercise 2. Consider the following two matrices:

$$A = \begin{pmatrix} 1 & -2 & 5 \\ 4 & -5 & 8 \\ -3 & 3 & -3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

(a) Find the reduced row-echelon form of the augmented matrix $[A \mid B]$.

(b) Is the system $AX = B$ is consistent (has a solution)? Justify your answer.

(c) Find the dimension of the solution space of the homogeneous system

$$AX = \mathbf{O}.$$

Exercise 3. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{pmatrix}.$$

(*i*) Use Gauss-Jordan Method to find the inverse of A.

(*ii*) Solve the system

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 5 \\ 2x_1 + 9x_2 + 3x_3 = -1 \\ x_1 + 4x_3 = 9 \end{cases}$$

Exercise 4. Let k be a real number and

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 0 & 1 & -1 \\ k & -2 & 0 \end{pmatrix}.$$

(a) Find all values of k for which A has a repeated eigenvalue.

(b) Suppose that $k = 1$.

- Find all eigenvalues of A and order them $\lambda_1 \leq \lambda_2 \leq \lambda_3$.
- Find an eigenvector of A corresponding to λ_2.