King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 302 Exam II

Semester (111) November 24, 2011 Time: 1:00 - 2:30 pm

Name: .....

I.D: ..... Section: .....

Exercise	Points
1	_____ 11
2	_____ 9
3	_____ 15
4	_____ 15
Total	_____ 50

Exercise 1.

(1) Let A be an $n \times n$-matrix, P be an $n \times n$ nonsingular (invertible) matrix and $D = P^{-1}AP$. Show that for each positive integer k, $A^k = PD^kP^{-1}$.

(2) Let $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$.

(i) Find a nonsingular matrix P that diagonalizes A.

(ii) Find $D = P^{-1}AP$ and A^{19}.

Exercise 2. Given the surface S defined by: $z = \sqrt{x^2 + y^2}$.

(i) Find the normal vector to S at the point $(1, 1, \sqrt{2})$.

(ii) Find an equation of the tangent plane to the surface at $(1, 1, \sqrt{2})$.

Exercise 3. Let $F = (-16y + \sin(x^2))\mathbf{i} + (4e^y + 3x^2)\mathbf{j}$ be a force acting along the positively oriented simple closed path $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \mathcal{C}_3$, where

- $\mathcal{C}_1$ is the positively oriented arc of the circle $x^2 + y^2 = 1$ with starting point $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and ending point $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$,
- $\mathcal{C}_2$ is the line segment joining $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and the origin $(0, 0)$.
- $\mathcal{C}_3$ is the line segment joining $(0, 0)$ and $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

(1) Graph the path $\mathcal{C}$.
(2) Evaluate the work done by the force F (remember Green's Theorem).

Exercise 4. Let $D = \{(x, y, z) \in \mathbb{R}^3 : x > 0, y > 0, z > 0\}$ and F be the vector field defined on D by

$$F(x, y, z) = \frac{1}{x}\,\mathbf{i} + \frac{1}{y}\,\mathbf{j} + \frac{1}{z}\,\mathbf{k}.$$

(1) Check that F is conservative.

(2) Evaluate the line integral $\int_{\mathcal{C}} F.dr$, where $\mathcal{C}$ is a path (piecewise smooth curve) in D with starting point $A = (1, 1, 1)$ and ending point $B = (1, 2, 1)$.