KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Department of Mathematics and Statistics

Math 301 Major Exam I

Thursday 13, October 2011

Duration 2 hrs

ID: ..... Section:.....

Name:.....

Exercise #1 (9 points)

Find the Laplace transform of

$$f(t) = 2t + \cos^2(3t)$$

Exercise #2 (9 points)

Find the inverse Laplace transform of

$$F(s) = \frac{s+7}{s^2+2s+5}$$

Exercise #3 (9 points)

Find the inverse Laplace transform of

$$G(s) = \frac{s-1}{s+1}\mathbf{e}^{-3s}$$

Exercise #4 (12 points)

Solve, using the Laplace transform, the initial value problem,

$$\begin{cases} y''(t) + 2y'(t) + y(t) = t\mathbf{e}^{-t} \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

Exercise #5 (12 points)

Solve, using the Laplace transform, the system

$$\begin{cases} x'(t) + x(t) - y(t) = 0 \\ 3x'(t) + y'(t) = \mathbf{e}^{-t} \\ x(0) = 0, \quad y(0) = 0 \end{cases}$$

`Do NOT solve the system of differential equations by reducing it into one single differential equation. NO MARK WILL BE GIVEN.`

Exercise #6 (10 points)

Given the vector function,

$$r(t) =< e^{2t}\sin(4t), e^{2t}\cos(4t), e^{2t} >$$

Find the length of the curve traced by the vector function when $0 \leq t \leq \frac{\pi}{8}$.

Exercise #7 (10 points)

(1) Find the directional derivative of $f(x, y, z) = 2x^3 - 3xy + yz$ at the point $P(2, 1, -1)$ in the direction of a unit vector whose angles with the $x-$, $y-$, and $z-$axes are, respectively, $-\frac{\pi}{4}$, $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(2) What is the maximum value of the directional derivative at $(1, 1, 1)$?

Exercise #8 (5 points)

Let $F = xy\mathbf{i} - \mathbf{x}^3yz\mathbf{j} + \mathbf{xy}^2\mathbf{k}$. Find $\operatorname{curl} F$ and $\operatorname{div} F$

Exercise #9 (12 points)

Find the work done by the force $F = y\mathbf{i} - x\mathbf{j} - z\mathbf{k}$ acting along the closed path $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$, where $\mathcal{C}_1$ is the helix

$r(t) = 2\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j} + t\mathbf{k}$, $0 \leq t \leq 2\pi$; and $\mathcal{C}_2$ is the segment that joins the endpoints of $\mathcal{C}_1$.

Exercise #10 (12 points)

Determine if the vector field $F = 2xz\mathbf{i}+2yz\mathbf{j} + (x^2 + y^2)\mathbf{k}$ is conservative. If so, find a potential ϕ for F. Then, find $\int_{\mathcal{C}} F \cdot dr$ where the path $\mathcal{C}$ is given by $r(t) = \sin(t)\mathbf{i}-t\mathbf{j} + \mathbf{e}^t\mathbf{k}$, $0 \leq t \leq 1$.