KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS

S111
FINAL EXAM MATH 301

NAME:..ID:...........................SECTION:......

Instructions:

- This exam is in two parts.
- Part I is of MCQ type and only the final answers (a, b, c, d, e) should be reported on this page.
- Part II is of written type.
- No formula sheet and/or calculator is allowed

MCQ type

Question #	1	2	3	4	5	6	7	Sub-Total
Answer								/42

Written Questions type

Question #	8	9	10	11	12	13	Sub-Total
Mark							/98

Total: /140

Exercise # 1: (6 pts) The directional derivative of

$$f(x, y, z) = 2\cos\left(\frac{\pi xy}{4}\right)\tan\left(\frac{\pi z}{2}\right)$$

at the point $(1, -1, \frac{1}{2})$ in the direction of the unit vector $< \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}} >$ is
(a) -2π
(b) $-\pi$
(c) π
(d) 2π
(e) none of the above

Exercise # 2: (6 pts) The Laplace transform of

$$f(t) = \int_0^t \sinh(\tau)\cosh(t-\tau)d\tau$$

is
(a) $F(s) = \frac{s}{(s^2+1)^2}$
(b) $F(s) = \frac{s}{(s^2-1)^2}$
(c) $F(s) = \frac{s}{(s^4-1)}$
(d) $F(s) = \frac{1}{(s^2-1)^2}$
(e) none of the above

Exercise # 3:(6 pts) Using the Laplace transform, the solution to the initial value problem

$$\begin{cases} y' + y = \delta(t-1) \quad , \quad t > 0 \\ y(0) = 1 \end{cases}$$

is (U being the unit step function and δ the Dirac delta function),
(a) $y = e^{-t}U(t) + U(t-1)$
(b) $y = e^{-t}U(t) + e^{-(t-1)}U(t-1)$
(c) $y = e^{-t}U(t) + t + e^{-(t-1)}U(t-1)$
(d) $y = e^{-t}U(t)$
(e) $y = (1+t)U(t) + e^{-(t-1)}U(t-1)$

Exercise # 4: (6 pts) The length of the curve traced by,

$$r(t) = e^{2t}\cos(3t)\mathbf{i} + e^{2t}\sin(3t)\mathbf{j} + e^{2t}\mathbf{k}$$

from $t = 0$ to $t = \frac{\pi}{2}$ is,
(a) $\frac{\sqrt{17}}{2}(e^{\pi} + 1)$
(b) $\frac{\sqrt{7}}{2}(e^{\pi} - 1)$
(c) $\frac{\sqrt{17}}{2}(e^{\pi} - 1)$
(d) $\frac{\sqrt{17}}{2}(e^{-\pi} - 1)$
(e) $\frac{7}{2}(e^{\pi} - 1)$

Exercise # 5:(6 pts) The work done by the force

$$F = e^x \cos\left(\frac{\pi y}{2}\right)\mathbf{i} - \frac{\pi}{2} e^x \sin\left(\frac{\pi y}{2}\right)\mathbf{j}$$

along the path C traced by $r(t) = t\mathbf{i} + t^2\mathbf{j}$ from $t = 0$ to $t = 2$. is
(a) $e^2 - 2$
(b) $e^2 + 1$
(c) $e^{-2} - 1$
(d) $e^2 + 2$
(e) $e^2 - 1$

Exercise # 6: (6 pts) Let D be the solid region in the first octant bounded by the planes $x = 1, y = 2$ and $z = 3$, and S its surface with unit normal $\mathbf{n}$. Let $F = xz^2 \cosh^2(z)\mathbf{i} - yz^2 \sinh^2(z)\mathbf{j} + 2xy\mathbf{k}$. Then, the surface integral

$$\int\int_S (F \cdot \mathbf{n})\, ds$$

is,
(a) 18
(b) 9
(c) -18
(d) -9
(e) none of the above
HINT: use the Divergence (Gauss) Theorem.

Exercise # 7: (6 pts) Both functions f_1 and f_2 defined by $f_1(x) = x^2$ and $f_2(x) = x^3$ are orthogonal to the function f_3 defined by $f_3(x) = 6 - x + c_1x^2 + c_2x^3$ on the interval $[-1, 1]$. Then the sum $c_1 + 5c_2$ is equal to

(a) 0
(b) -3
(c) -1
(d) 1
(e) 3

Exercise # 8:(8 pts) Determine the regions in the xy-plane for which the equation

$$y\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} + xyu = 0$$

is parabolic, hyperbolic or elliptic.

Exercise # 9: (19 pts) Consider the Sturm-Liouville problem,

$$\begin{cases} y'' + y' + \lambda y = 0 \quad , \quad 0 < x < 1 \\ y(0) = 0 = y(1) \end{cases}$$

(a) Write the differential equation in self adjoint form.
(b) Say whether the problem is regular, periodic or singular
(c) Find the weight function
(d) Find the eigenvalues and corresponding eigenfunctions.
(e) Write the form of the eigenfunctions expansion of the function given by $f(x) = x$ as well as its coefficients (do not evaluate).

Exercise # 10: (19 pts) Use the method of separation of variables to solve

$$\begin{cases} \frac{\partial u}{\partial t} = 81\frac{\partial^2 u}{\partial x^2} \quad , \quad 0 < x < 1 \ , \ t > 0 \\ u(0,t) = 0 = u(1,t) \ , \ t \geq 0 \\ u(x,0) = 2\sin(\pi x) - 7\sin(5\pi x) \quad , \ 0 < x < 1 \end{cases}$$

Exercise # 11: (19 pts) Use the Laplace Transform to solve

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2} \ , \ 0 < x < \pi \ , \ t > 0 \\ u(0,t) = 0 = u(\pi,t) \ , \ t \geq 0 \\ u(x,0) = 0 \ , \ 0 < x < \pi \\ u_t(x,0) = 4\sin(3x) \ , \ 0 < x < \pi \end{cases}$$

Exercise # 12: (14 pts) Find the Fourier integral representation of the function f defined by,

$$f(x) = \begin{cases} x+1\ , & -1 < x < 0 \\ 1\ , & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Exercise # 13:(19 pts) Use the Fourier transform to solve

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \ , \ 0 < x < 1 \ , \ -\infty < y < +\infty \\ u(0, y) = 0 \\ u(1, y) = f(y) \ , \ -\infty < y < +\infty \end{cases}$$

DO NOT EVALUATE THE RESULTING INTEGRAL.