version 01

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 260
Exam II, Semester I, 2011-2012
Net Time Allowed: 120 minutes

Name:____

ID:____________________ Section:__________ Serial:____________

Q#	Marks	Maximum Marks
1-8		24
9		7
10		6
11		7
12		4
13		17
Total		65

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

Note:
You should write all your answers of part I in the box below.

Part I

1	2	3	4	5	6	7	8	9(a)	9(b)	9(c)	9(d)	9(e)	9(f)	9(g)

Part I

1. If $A = \begin{bmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix}$ and $A^2 + \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} = I_3$, then $x + y + z =$

   (a) $\frac{9}{8}$

   (b) $\frac{-7}{8}$

   (c) $\frac{-9}{8}$

   (d) $\frac{7}{8}$

   (e) 1

2. If $| A |= 5$ where A is 3×3 matrix, then $| (2A)^{-1} |=$

   (a) $\frac{-1}{40}$

   (b) $\frac{1}{40}$

   (c) $\frac{1}{10}$

   (d) $\frac{1}{-10}$

   (e) $\frac{2}{5}$

3. Let E_1 be the 3×3 elementary matrix that multiplies row 1 by 3, let E_2 be the 3×3 elementary matrix that interchange row 2 by row 3, and let E_3 be the 3×3 elementary matrix that adds two times row 2 to row 1. If determinant of A equals 10, and $C = E_1 E_2 E_3 A$. The value of the determinant of C is equal to:

   (a) -30

   (b) 30

   (c) -60

   (d) 60

   (e) Can not be computed

4. Let $S = \{A_1, A_2, A_3\}$, where

$$A_1 = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}.$$

   Then

   (a) S span the vector space $M_{2\times 2}(\mathbb{R})$

   (b) S form a basis for vector space $M_{2\times 2}(\mathbb{R})$

   (c) S is linearly independent only.

   (d) S is linearly dependent.

   (e) All choices are wrong.

5. If $\begin{bmatrix} -1 \\ -0.5 \\ 0 \end{bmatrix}_\beta = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ relative to the basis β of $\mathbb{R}^3$, where $\beta = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}$, then $a + b + c$ is equal to:

(a) 1

(b) -1

(c) 2

(d) -2

(e) 0

6. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 6$, then $\begin{vmatrix} 2a & 2d & 2g \\ b-c & e-f & h-i \\ c-a & f-d & i-g \end{vmatrix}$ is equal to:

(a) 0

(b) 12

(c) 6

(d) 48

(e) 18

7. The homogeneous DE with $0, 1, 1, 2-3i$ as roots of its characteristic equation is :

(a) $y^{(5)} - 6y^{(4)} + 22y''' - 40y'' + 13y = 0.$

(b) $y^{(5)} - 6y^{(4)} + 22y''' - 30y'' - 13y' = 0.$

(c) $y^{(5)} - 6y^{(4)} + 22y''' - 30y'' + 13y' = 0.$

(d) $y^{(4)} - (4-3i)y^{(3)} + (5-6i)y'' - (2-3i)y' = 0.$

(e) $y^{(4)} + (4-3i)y^{(3)} + (5-6i)y'' + (2-3i)y' = 0.$

8. The particular solution of the second order homogeneous DE

$$y'' - 6y' + 4y = 0, \qquad y(0) = 0, y'(0) = \sqrt{10}$$

at $x = 1$ is:

(a) $y = \frac{\sqrt{10}}{3}(e^4 - e)$

(b) $y = e^{(3+\sqrt{5})} - e^{(3-\sqrt{5})}$

(c) $y = e^{(5+\sqrt{3})} + e^{(5-\sqrt{3})}$

(d) $y = \frac{1}{\sqrt{2}}(e^{(5+\sqrt{3})} - e^{(5-\sqrt{3})})$

(e) $y = \frac{1}{\sqrt{2}}(e^{(3+\sqrt{5})} - e^{(3-\sqrt{5})})$

9. **Answer TRUE** ($\surd$) **or FALSE** ($\times$).

(a) Let A and B be $n \times n$ matrices, then $(A+B)^2 = A^2 + 2AB + B^2$.

(b) If C is an invertible matrix and $AC = CB$, then $A = B$.

(c) If the linear system $Ax = 0$ has a nontrivial solution, then A can be expressed as a product of elementary matrices.

(d) If A is an $n \times n$ matrix, then $\det(A^2)$ can not be negative.

(e) Let A be an $n \times n$ matrix and c be a scalar, then $(cA)^{-1} = \dfrac{1}{c}A^{-1}$

(f) Let A be an $n \times n$ matrix, then $A^T - A$ is a symmetric matrix.

(g) If $S = \{A \in M_{2\times 2}(\mathbb{R}) : \det(A) = 0\}$. Then S is a vector space.

Part II

10. Using Cramer's rule find the value of x_3 in the following system

$$\begin{aligned} x_1 + x_2 - x_3 &= 6 \\ x_1 - x_2 + x_3 &= 2 \\ x_1 - 2x_3 &= 0 \end{aligned}$$

11. Using the adjoint matrix find the inverse of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 6 \\ 0 & -4 & 12 \end{bmatrix}$$

12. Let A be $n \times n$ matrix with $A^4 = 0$. Prove that $I_n - A$ is invertible with

$$(I_n - A)^{-1} = I_n + A + A^2 + A^3.$$

13. In $\mathbb{R}^4$, let W be the subspace of all vectors $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$ that satisfy $a_4 - a_3 = a_2 - a_1$.

(a) Show that W is a subspace of $\mathbb{R}^4$.

(b) Show that $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ span W.

(c) Find a subset α of S that is a basis for W.

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