

n Section 1.1 :

In Problems 1 through 12, verify by substitution that each given function is a solution of the given differential equation. Throughout these problems, primes denote derivatives with respect to x .

2. $y' + 2y = 0; y = 3e^{-2x}$

12. $x^2y'' - xy' + 2y = 0; y_1 = x \cos(\ln x), y_2 = x \sin(\ln x)$

In Problems 17 through 26, first verify that $y(x)$ satisfies the given differential equation. Then determine a value of the constant C so that $y(x)$ satisfies the given initial condition. Use a computer or graphing calculator (if desired) to sketch several typical solutions of the given differential equation, and highlight the one that satisfies the given initial condition.

22. $e^y y' = 1; y(x) = \ln(x + C), y(0) = 0$

In Problems 27 through 31, a function $y = g(x)$ is described by some geometric property of its graph. Write a differential equation of the form $dy/dx = f(x, y)$ having the function g as its solution (or as one of its solutions).

30. The graph of g is normal to every curve of the form $y = x^2 + k$ (k is a constant) where they meet.

36. In a city with a fixed population of P persons, the time rate of change of the number N of those persons infected with a certain contagious disease is proportional to the product of the number who have the disease and the number who do not.

In Problems 37 through 42, determine by inspection at least one solution of the given differential equation. That is, use your knowledge of derivatives to make an intelligent guess. Then test your hypothesis.

40. $(y')^2 + y^2 = 1$

n Section 1.2 :

In Problems 1 through 10, find a function $y = f(x)$ satisfying the given differential equation and the prescribed initial condition.

~~4~~ $\frac{dy}{dx} = \frac{1}{x^2}; y(1) = 5$

~~6~~ $\frac{dy}{dx} = x\sqrt{x^2 + 9}; y(-4) = 0$

In Problems 11 through 18, find the position function $x(t)$ of a moving particle with the given acceleration $a(t)$, initial position $x_0 = x(0)$, and initial velocity $v_0 = v(0)$.

~~15~~ $a(t) = 4(t + 3)^2, v_0 = -1, x_0 = 1$
 $(t + 1)^2$

~~18~~ $a(t) = 50 \sin 5t, v_0 = -10, x_0 = 8$

n Section 1.4 :

Find general solutions (implicit if necessary, explicit if convenient) of the differential equations in Problems 1 through 18. Primes denote derivatives with respect to x .

9. $\frac{dy}{dx} + 2xy = 0$ 10. $(1+x)^2 \frac{dy}{dx} = (1+y)^2$

Find explicit particular solutions of the initial value problems in Problems 19 through 28.

24. $(\tan x) \frac{dy}{dx} = y, \quad y\left(\frac{1}{2}\pi\right) = \frac{1}{2}\pi$ 27. $\frac{dy}{dx} = 6e^{2x-y}, \quad y(0) = 0$

33. (Population growth) A certain city had a population of 25,000 in 1960 and a population of 30,000 in 1970. Assume that its population will continue to grow exponentially at a constant rate. What population can its city planners expect in the year 2000?

n Section 1.5 :

Find general solutions of the differential equations in Problems 1 through 25. If an initial condition is given, find the corresponding particular solution. Throughout, primes denote derivatives with respect to x .

4. $y' - 2xy = e^{x^2}$ 12. $xy' + 3y = 2x^5, y(2) = 1$

24. $(x^2 + 4)y' + 3xy = x, y(0) = 1$

Solve the differential equations in Problems 26 through 28 by regarding y as the independent variable rather than x .

28. $(1 + 2xy) \frac{dy}{dx} = 1 + y^2$

32. (a) Find constants A and B such that $y_p(x) = A \sin x + B \cos x$ is a solution of $dy/dx + y = 2 \sin x$. (b) Use the result of part (a) and the method of Problem 31 to find the general solution of $dy/dx + y = 2 \sin x$. (c) Solve the initial value problem $dy/dx + y = 2 \sin x, y(0) = 1$.

n Section 1.6 :

Find general solutions of the differential equations in Problems 1 through 30. Primes denote derivatives with respect to x throughout.

2. $2xyy' = x^2 + 2y^2$ 10. $xyy' = x^2 + 3y^2$

22. $x^2y' + 2xy = 5y^4$

In Problems 31 through 42, verify that the given differential equation is exact; then solve it.

40. $(e^x \sin y + \tan y) dx + (e^x \cos y + x \sec^2 y) dy = 0$

59. Solve the differential equation

$$\frac{dy}{dx} = \frac{x - y - 1}{x + y + 3}$$

by finding h and k so that the substitutions $x = u + h$, $y = v + k$ transform it into the homogeneous equation

$$\frac{dv}{du} = \frac{u - v}{u + v}.$$

60. Use the method in Problem 59 to solve the differential equation

$$\frac{dy}{dx} = \frac{2y - x + 7}{4x - 3y - 18}.$$