Version 1

King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 260

Final Exam, Second Semester (111), 2011-2012

Net Time Allowed: 180 minutes

Name:____

ID:____________________Section:__________Serial:__________

Q#	Marks	Maximum Marks
1		6
2		6
3		7
4		8
5		4
6		9
7-18		60
Total		100

Part II

7	8	9	10	11	12	13	14	15	16	17	18
c	e	d	a	d	c	a	e	b	c	d	e

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

1. Write the following system of DE as a system of first order linear DE in matrix form

$$
\begin{aligned}
y_1^{''} &= y_1^{'} + y_2 + e^x \\
y_2^{(3)} &= y_1 + y_2 + y_2^{''} + \sin(x)
\end{aligned}
$$

2. Given the charactersitic polynomial $p(\lambda) = \lambda(\lambda^3 - 3\lambda^2 + 3\lambda - 1)$ of a matrix A. Write all possible forms of the Jordan form of A

3. Verify that the set of all real solutions to the following system is a subspace of $\mathbb{R}^3$:

$$\begin{aligned} x_1 - 3x_2 - x_3 &= 0, \\ x_1 - 2x_2 - 4x_3 &= 0. \end{aligned}$$

4. Solve

$$\frac{dY}{dt} = \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix} Y$$

Hint: The eigenvalues are: $\lambda = \mp i$.

5. If an $n \times n$ matrix A is similar to $n \times n$ matrix B. Show that A and B have same eigenvalues

6. Find a particular solution of

$$y'' - 2y' + y = \frac{e^t}{t^2 + 1}$$

7. The general solution of the system of DE

$$Y' = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 3 \\ 0 & 0 & 3 \end{bmatrix} Y, \qquad \text{is}$$

(a) $\begin{bmatrix} 0 & e^{3t} & -\frac{2}{3}e^{3t} \\ e^{2t} & 0 & te^{3t} \\ e^{2t} & 0 & \frac{1}{3}e^{3t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

(b) $\begin{bmatrix} e^{2t} & 0 & -\frac{2}{3}e^{3t} \\ 0 & e^{3t} & 0 \\ 0 & 0 & \frac{1}{3}e^{3t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

(c) $\begin{bmatrix} e^{2t} & 0 & -\frac{2}{3}e^{3t} \\ 0 & e^{3t} & te^{3t} \\ 0 & 0 & \frac{1}{3}e^{3t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

(d) $\begin{bmatrix} e^{2t} & 0 & -\frac{2t}{3}e^{3t} \\ 0 & e^{3t} & e^{3t} \\ 0 & 0 & \frac{t}{3}e^{3t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

(e) $\begin{bmatrix} e^{2t} & 0 & -\frac{2}{3}e^{3t} \\ 0 & e^{3t} & e^{3t} \\ 0 & 0 & \frac{1}{3}e^{3t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

8. For what value(s) of a the inverse of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 2 & a \end{bmatrix}$$

exists

(a) $a = -1$
(b) $a \neq 0$
(c) $a \neq 1$
(d) $a = 1$
(e) $a \neq -1$

9. Let $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & 0 & 1 \end{bmatrix}$, $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}$ and $E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be elementry matrices such that

$$E_4 \times E_3 \times E_2 \times E_1 \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Then $a + b + c + d =$

(a) 0

(b) -9

(c) $-\frac{47}{4}$

(d) $-\frac{37}{4}$

(e) $\frac{1}{2}$

10. Let $S = \{P_1, P_2, P_3\}$ where $P_1 = 2 + x + 4x^2$, $P_2 = 1 - x - 3x^2$ and $P_3 = 4 + 2x + 5x^2$. Write $5 + x + 6x^2$ as a linear combination of vectors in S, then the sum of these constants equal to

(a) $\frac{10}{3}$

(b) $-\frac{10}{3}$

(c) 0

(d) $\frac{5}{3}$

(e) $-\frac{5}{3}$

11. Let $S = \{v_1, v_2, v_3\}$ where $v_1 = (1, 2, 1),\ v_2 = (2, 9, 0)$ and $v_3 = (3, 3, 4)$. Then

(a) S is linearly indepedent but do not span $\mathbb{R}^3$

(b) S span $\mathbb{R}^3$ and S is linearly dependent

(c) S do not span $\mathbb{R}^3$ and S is linearly dependent

(d) S form a basis for $\mathbb{R}^3$

(e) All are wrong.

12. The solution of the Bernoulli DE

$$y^{-2}\frac{dy}{dx} + \frac{4}{x}y^{-1} = x^3, \quad y(e) = e^{-4}$$

at $x = 2$ is

(a) $\dfrac{1}{2^5 + 2^4 \ln 2}$

(b) $\dfrac{1}{2^4 - 2^5 \ln 2}$

(c) $\dfrac{1}{2^5 - 2^4 \ln 2}$

(d) $\dfrac{1}{2^4 + 2^5 \ln 2}$

(e) e^2

13. The general solution of the DE

$$y''' + 5y'' + 8y' + 4y = 0,$$

is

(a) $y = c_1e^{-x} + c_2e^{-2x} + c_3xe^{-2x}$
(b) $y = c_1e^{-x} + c_2e^{-4x} + c_3e^{-2x}$
(c) $y = c_1e^{-x} + c_2e^{-2x}$
(d) $y = c_1e^{x} + e^{-2x}(c_2\cos x + c_3\sin x)$
(e) $y = c_1e^{x} + c_2e^{2x} + c_3e^{-2x}$

14. The most general form of a particular solution of the non-homogeneous part which does not satisfy the homogeneous part associated with the DE

$$y^{(4)} + y''' = 1 - x^2e^{-x}$$

is

(a) $y = Ax^3 + (Bx^2 + Cx + D)e^{-x}$
(b) $y = Ax^3 + (Bx^4 + Cx^3 + Dx^2)e^{-x}$
(c) $y = A + (Bx^2 + Cx + D)e^{-x}$
(d) $y = (Ax + E) + (Bx^2 + Cx + D)e^{-x}$
(e) $y = Ax^3 + (Bx^3 + Cx^2 + Dx)e^{-x}$

15. The value of the solution of the DE

$$y'' + 4y = 12x, \qquad y(0) = 5,\ y'(0) = 7,$$

at $x = \frac{\pi}{8}$ is

(a) $\dfrac{3\pi}{2} - \dfrac{7}{\sqrt{2}}$

(b) $\dfrac{3\pi}{8} + \dfrac{7}{\sqrt{2}}$

(c) $\dfrac{3\pi}{8} + \dfrac{9}{2\sqrt{2}}$

(d) $\dfrac{3\pi}{8} - \dfrac{9}{2\sqrt{2}}$

(e) $\dfrac{9}{2\sqrt{2}}$

16. The sum of the squares of the eigenvalues $(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)$ of the following matrix

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

(a) 16

(b) 9

(c) 14

(d) 25

(e) 36

17. The solution for the following system of DE

$$\frac{dX}{dt} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} X,$$

Hint: The eigenvalues are $0, 1, 2$

(a) $X = \begin{bmatrix} 0 & e^t & e^{2t} \\ -1 & 0 & 0 \\ 1 & 0 & e^{2t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

(b) $X = \begin{bmatrix} 0 & e^t & 0 \\ -1 & 0 & e^{2t} \\ 1 & e^{2t} & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

(c) $X = \begin{bmatrix} 0 & e^t & 0 \\ -1 & 0 & te^{2t} \\ 1 & 0 & te^{2t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

(d) $X = \begin{bmatrix} 0 & e^t & 0 \\ -1 & 0 & e^{2t} \\ 1 & 0 & e^{2t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

(e) $X = \begin{bmatrix} -t & e^t & 0 \\ -t & e^t & e^{2t} \\ t & 0 & e^{2t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

18. The particular solution of the exact DE

$$(\cos x - x \sin x + y^2)dx + (2xy)dy = 0, \qquad y(\pi) = 1. \quad \text{is}$$

(a) $xy^2 + x\cos x + \sin x = 0$

(b) $xy^2 + \cos x = 0$

(c) $xy^2 + x\sin x = 0$

(d) $y^2 + x\cos x = 0$

(e) $xy^2 + x\cos x = 0$