December 21,2011

QUIZ♯2 Math202 **Time allowed: 15 minutes**

Name: **ID ♯ :** **Serial:**

Exercise: (10pts).

Let

$$A = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$X_1 = \begin{pmatrix} 1 \\ -1-\sqrt{2} \end{pmatrix} e^{\sqrt{2}t}, X_2 = \begin{pmatrix} 1 \\ -1+\sqrt{2} \end{pmatrix} e^{-\sqrt{2}t} \text{ and } X_p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t^2 + \begin{pmatrix} -2 \\ 4 \end{pmatrix} t + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Prove that the general solution of

$$X' = AX + \begin{pmatrix} 1 \\ 1 \end{pmatrix} t^2 + \begin{pmatrix} 4 \\ -6 \end{pmatrix} t + \begin{pmatrix} -1 \\ 5 \end{pmatrix} \qquad (1)$$

is $X = c_1 X_1 + c_2 X_2 + \quad X_p$
solution.

• $X_p' = \begin{pmatrix} 2 \\ 0 \end{pmatrix} t + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2t-2 \\ 4 \end{pmatrix}$ (0.1)

• $AX_p + \begin{pmatrix} t^2+4t-1 \\ t^2-6t+5 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} t^2-2t+1 \\ 4t \end{pmatrix} + \begin{pmatrix} t^2+4t-1 \\ t^2-6t+5 \end{pmatrix} = \begin{pmatrix} -t^2-2t-1 \\ -t^2+6t-1 \end{pmatrix} + \begin{pmatrix} t^2+4t+1 \\ t^2-6t+5 \end{pmatrix}$

$= \begin{pmatrix} 2t-2 \\ 4 \end{pmatrix} = X_p'$ (0.2)

Thus X_p is a particular solution of the system (1). (0.1)

• Now $X_1' = \begin{pmatrix} \sqrt{2} \\ -\sqrt{2}-2 \end{pmatrix} e^{\sqrt{2}t}$ (0.1) ; $AX_1 = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} e^{\sqrt{2}t} \\ -e^{\sqrt{2}t} - \sqrt{2}e^{\sqrt{2}t} \end{pmatrix}$ (0.1) $= \begin{pmatrix} \sqrt{2}e^{\sqrt{2}t} \\ -2e^{\sqrt{2}t} - \sqrt{2}e^{\sqrt{2}t} \end{pmatrix} = X_1'$

Thus X_1 is a solution (0.5) of the Homogeneous system: $X' = AX$.

• $X_2' = \begin{pmatrix} -\sqrt{2} \\ \sqrt{2}-2 \end{pmatrix} e^{-\sqrt{2}t}$ (0.1) $= AX_2 = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} e^{-\sqrt{2}t} \\ -e^{-\sqrt{2}t} + \sqrt{2}e^{-\sqrt{2}t} \end{pmatrix}$ (0.1)

$\Rightarrow$ X_2 is a solution (0.5) of the homogeneous system $X' = AX$
and X_1, X_2 are Linearly independent (clear) (0.5)
Hence the general solution is given by:

$\boxed{X = c_1 X_1 + c_2 X_2 + X_p}$ (0.5)