

Name:

ID #:

Serial:

Exercise1: (7pts)Solve the DE:  $\cos^2 x \sin x \frac{dy}{dx} + \cos^3 x y = 1$ 

$$\text{So F: } \frac{dy}{dx} + \frac{\cos x}{\sin x} y = \frac{1}{\cos^2 x \sin x} \Rightarrow \text{I.F: } e^{\int \frac{\cos x}{\sin x} dx} = e^{\ln|\sin x|} = \sin x$$

$$\text{Thus: } \sin x \frac{dy}{dx} + \cos x y = \frac{1}{\cos^2 x} = \frac{d}{dx} [\sin x y] = \frac{d}{dx} (\tan x)$$

$$\text{Hence } \sin x y = \tan x + C \Rightarrow y = \frac{1}{\cos x} + \frac{C}{\sin x}$$

$$\Rightarrow \boxed{y = \sec x + C \csc x}$$

Exercise2: (6pts)

Solve the initial value problem:

$$(xy - 2x) dx - (xy + y) dy = 0, y(0) = 3.$$

$$x(y-2) dx - y(x+1) dy = 0: \text{Dividing both sides by } (y-2)(x-1) \Rightarrow$$

$$\frac{x}{x+1} dx - \frac{y}{y-2} dy = 0 = \left(1 - \frac{1}{x+1}\right) dx - \left(1 + \frac{2}{y-2}\right) dy \xrightarrow{\text{Integr.}}$$

$$x - \ln|x+1| - y - 2 \ln|y-2| = C$$

$$\text{Now } y(0) = 3 \Rightarrow C = -3$$

$$\text{Hence } \boxed{x - \ln|x+1| - y - 2 \ln|y-2| = -3}$$