MASTER

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 202 Final Exam
The First Semester of 2011-2012 (111)
Time Allowed: 180 Minutes

Name: ______________________ ID#: ______________

Section/Instructor: ______________ Serial #: ______________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.
- Write MCQ Answers on the front page.

Written Questions

Question #	Marks	Maximum Marks
1		12
2		20
3		16
4		20
5		16
Total		/84

Multiple Choice Questions

Question #	Student Answer	Marks	Maximum Marks
1			7
2			7
3			7
4			7
5			7
6			7
7			7
8			7
Total			/56
Grand Total			/140

Q:1 (12 pints) Determine singular points of the differential equation

$$x^3(x^2-16)(x-1)^2y'' + 3x(x-1)y' + 5(x+4)y = 0.$$

Classify each singular point as regular or irregular.

Q:2 (20 points) Find two linearly independent power series solutions of $y'' + xy' + 3y = 0$ about the ordinary point $x = 0$. Give the first three nonzero terms for each series solution.

Q:3 (16 points) Solve the differential equation $X' = AX$, where $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

Q:4 (20 points) Solve the initial value problem $X' = AX$, $X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ where $A = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$

Q:5 (16 points) Find X_p for the system $X' = AX + F(t)$, where $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$, $F(t) = \begin{bmatrix} 70e^{4t} \\ 20e^{t} \end{bmatrix}$ and $X_c = c_1 \begin{bmatrix} e^{-3t} \\ -2e^{-3t} \end{bmatrix} + c_2 \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix}$.

1. Integrating factor that makes the differential equation $(-xy\sin x + 2y\cos x)dx + 2x\cos x dy = 0$ EXACT is:

(a) $(\sec x)^{\frac{1}{2}}$

(b) $(\tan x)^{\frac{1}{2}}$

(c) $\frac{1}{2}\ln(\sec x)$

(d) $\frac{1}{2}\sec x$

(e) $\frac{1}{2}\tan x$

2. If $y(x)$ is a solution of $(x + ye^{\frac{y}{x}})dx - xe^{\frac{y}{x}}dy = 0$ with $y(1) = 0$, then $y(e^2)$ is equal to:

(a) $e^2 \ln 3$

(b) $e \ln 3$

(c) $3 \ln 3$

(d) $e \ln 9$

(e) $e^2 \ln 9$

3. After 6 hours, a radioactive material has decreased by 87.5% (remained 12.5%). What is the half life of the material?

(a) 2 hours

(b) 4 hours

(c) 6 hours

(d) 8 hours

(e) ∞ hours

4. If $y(x)$ is the solution of the initial value problem $3y''' + 2y'' = 0$, $y(0) = -1,\ y'(0) = 0,\ y''(0) = 1$, then $y(\frac{3}{2})$ is equal to:

(a) $\dfrac{9}{4e} - 1$

(b) $\dfrac{3}{4e} - 1$

(c) $\dfrac{3}{4e} + 1$

(d) $\dfrac{9}{4e} + 1$

(e) $\dfrac{3}{2e} - 1$

5. Which one of the following functions is annihilated by the operator $(D+1)(D^2-6D+25)$

(a) $e^{-x}+e^{3x}\cos 4x$

(b) $e^{-x}+xe^{3x}\cos 4x$

(c) $e^{-x}+e^{5x}+xe^{5x}$

(d) $e^{-x}+e^{-5x}+xe^{-5x}$

(e) $e^{-x}+e^{-3x}\cos 5x+e^{-3x}\sin 5x$

6. If $y_c = c_1\cos 2x + c_2\sin 2x$ is complementary function of the equation $4y''+16y=\csc 2x$, then a particular solution is given by:

(a) $-\frac{1}{8}x\cos 2x+\frac{1}{16}\sin 2x\ln|\sin 2x|$

(b) $-\frac{1}{16}x\cos 2x+\frac{1}{8}\sin 2x\ln|\sin 2x|$

(c) $-\frac{1}{8}x\cos 2x-\frac{1}{8}\sin 2x\ln|\sec 2x|$

(d) $-\frac{1}{8}x\cos 2x+\frac{1}{16}\sin 2x\ln|\cos 2x|$

(e) $-\frac{1}{16}x\cos 2x+\frac{1}{8}\sin 2x\ln|\csc 2x|$

7. If we convert the Cauchy-Euler equation $-2x^2y'' + xy' - 2y = 0$ into an equation with constant coefficients $y'' + ay' + by = 0$, then $a + b$ is equal to:

(a) $-\frac{1}{2}$

(b) $-\frac{3}{2}$

(c) $\frac{1}{2}$

(d) $-\frac{1}{3}$

(e) $-\frac{2}{3}$

8. If $y_{p_1} = xe^x$ is a particular solution of $y'' - y' = e^x$
and $y_{p_2} = \frac{1}{2}(\cos x - \sin x)$ is a particular solution of $y'' - y' = \sin x$,
then $y_p = \sin x - \cos x - xe^x$ is a particular solution of:

(a) $y'' - y' = -2\sin x - e^x$

(b) $y'' - y' = \sin x - \cos x + e^x$

(c) $y'' - y' = 2\sin x - e^x$

(d) $y'' - y' = e^x - \cos x$

(e) $y'' - y' = 2e^x - 2\sin x$