King Fahd University of Petroleum & Minerals Department of Mathematics & Statistics Math 202 Major Exam 2 The First Semester of 2011-2012 (111)

 $\mathbf{Q:1}$ Consider the differential equation

$$y'' + 3y = -18e^{3x} \tag{1}$$

- (a) (5 points) Verify that $y_1 = \cos \sqrt{3}x$ and $y_2 = \sin \sqrt{3}x$ are solutions of y'' + 3y = 0.
- (b) (5 points) Find a particular solution of the differential equation (1).
- (c) (2 points) Write the general solution of the differential equation (1).

Q:2 (16 points) Find the general solution of the differential equation

$$xy'' - y' + 4x^3y = 0$$

given that $y_1 = \sin(x^2)$ is a solution.

Q:3 (16 points) Solve the initial value problem

$$y''' + y'' - y' - y = 0;$$
 $y(0) = 0, y'(0) = 0, y''(0) = 4.$

Q:4 (a) (8 points) Find a linear differential operator that annihilates the function

$$9x^3 + x^2(1 - 3x)e^{2x} + 5xe^{-2x}\cos 3x$$

(b) (16 points) Solve the differential equation $y'' - 5y' + 6y = 3e^x + 2\sin x$ using undetermined coefficients.

Q:5 (a) (12 points) Use variation of parameters method to find a particular solution of

$$(x^{2}-1)y''-2xy'+2y = (x^{2}-1)^{2}$$

given that a complementary function is $y_c = c_1 x + c_2 (1 + x^2)$.

(b) (4 points) Write the general solution and find values of c_1 and c_2 using the boundary conditions y(0) = 0 and y(1) = 1.

 ${\bf Q:6}$ (16 points) Find the general solution of

$$4x^2y'' + 8xy' + y = 4x^{\frac{3}{2}}$$