King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 202   Major Exam I
The First Semester of 2011-2012 (111)
<u>**Time Allowed: 120 Minutes**</u>

Name: _____   ID#: _____

Section/Instructor: _____   Serial #: _____

- Mobiles and calculators are not allowed in this exam.

- Write all steps clear.

Question #	Marks	Maximum Marks
1		10
2		10
3		12
4		15
5		15
6		15
7		15
8		8
Total		100

Q:1 (a) (5 points) Verify that $y = e^{-x^2} \int\limits_0^x e^{t^2}\, dt + c_1 e^{-x^2}$ is a one parameter family of solutions of

$$2y' + 4xy = 2.$$

(b) (5 points) Find values of m such that $y = e^{mx}$ is a solution of $y'' - 7y' + 12y = 0$.

Q:2 (a) (5 points) Find value of c such that $y = \dfrac{1}{x^2 + c}$ is a solution of the initial value problem $y' + 2xy^2 = 0$, $y(0) = -1$.

(b) (3 points) Give intervals of definition of y.

(c) (2 points) Give the largest interval over which the solution is defined.

Q:3 (12 points) Solve the IVP

$$(y^2 + 1)dx = y \sec^2 x \, dy \qquad y(0) = 1.$$

Write the solution in explicit form.

Q:4 (15 points) Solve the linear differential equation

$$dx = (\sin 2y + x \tan y) \, dy.$$

Q:5 (15 points) Solve the initial value problem

$$(\sin y - y \sin x)\, dx + (1 + \cos x + x \cos y)\, dy = 0, \quad y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}.$$

Q:6 (a) (8 points) Use a suitable substitution to find a linear differential equation from

$$y^{\frac{3}{2}} dy = (2x - y^{\frac{5}{2}}) dx.$$

<u>Do NOT</u> solve the new equation.

(b) (7 points) Use a suitable substitution to find a separable differential equation from

$$\frac{dy}{dx} = \frac{y^2 - x^2}{xy}.$$

<u>Do NOT</u> solve the new equation.

Q:7 (15 points) Coffee is poured from a pot whose contents are at 95° degrees into a cup that is placed in a room whose ambient temperature is 20° degrees. After one minute the coffee has cooled down to 90° degrees. How long will it take the coffee temperature to reach 65° degrees?

Q:8 (a) (8 points) Make the following <u>Nonexact</u> differential equation

$$2xy\ dx + (3x^2 + y^2 + 5)\ dy = 0$$

an <u>EXACT</u> differential equation using an appropriate integrating factor. <u>Do NOT</u> solve the new equation.