October 29,2011

Test♯2 Math202, sec 5, Time allowed: 45 minutes

Name: ID ♯ : Serial:

Exercise1: (06pts)

Solve the DE: $\cos^2 x \sin x \frac{dy}{dx} + (\cos^3 x)y = 1$

S.F: $\frac{dy}{dx} + \frac{\cos x}{\sin x} y = \frac{1}{\cos^2 x \sin x}$ 02 $\Rightarrow$ I.F: $e^{\int \frac{\cos x}{\sin x}dx} = e^{\ln|\sin x|} = \sin x$ 01 02

Thus $\sin x \frac{dy}{dx} + \cos x\, y = \frac{1}{\cos^2 x} \Rightarrow \frac{d}{dx}[\sin x\, y] = \frac{d}{dx}(\tan x)$

Hence $\sin x\, y = \tan x + C$

$\Rightarrow \boxed{y = \sec x + C \csc x}$ 01

Exercise2: (07)

Solve the initial value problem:

$$(xy - 2x)dx - (xy + y)dy = 0 \ \ , y(0) = 3.$$

$x(y-2)dx - y(x+1)dx = 0 \Rightarrow \frac{x}{x+1}dx - \frac{y}{y-2}dy = 0$ 02

$\Rightarrow \left(1 - \frac{1}{x+1}\right)dx - \left(1 + \frac{2}{y-2}\right)dy = 0$ 01, Integrating both sides:

$x - \ln|x+1| - y - 2\ln|y-2| = C$ 02; substituting now $y(0)=3$

we find $\boxed{C = -3}$ 01 Hence $\boxed{x - \ln|x+1| - y - 2\ln|y-2| = -3}$ is the solution of the given IVP, 01

Exercise3: (07pts)

Find the general solution of the initial value problem:

$$y'' + 2y' + 2y = 0, \quad y(\frac{\pi}{4}) = 2, \ y'(\frac{\pi}{4}) = -2$$

Aux. Eq: $m^2 + 2m + 2 = 0$ 01, the roots are $m_1 = -1+i$ and $m_2 = -1-i$ 01

thus $y = e^{-x}(C_1 \cos x + C_2 \sin x)$ 02, now $y(\frac{\pi}{4}) = 2$ and $y'(\frac{\pi}{4}) = -2 \Rightarrow$

$\boxed{C_1 = C_2 = \sqrt{2}\, e^{\frac{\pi}{4}}}$ 02 Hence $\boxed{y = \sqrt{2}\, e^{\frac{\pi}{4}} (\cos x + \sin x) e^{-x}}$ 01