October 3,2011

Test♯1 Math202, sec 5, **Time allowed: 35 minutes**

Name: **ID ♯ :** **Serial:**

Exercise1: (07pts)

Solve the DE: $2xy\,\frac{dy}{dx} = 3\,y^2 + 4x^2$

$(4x^2+3y^2)\,dx - 2xy\,dy = 0$, This is a Homogeneous (DE) with order 2. (01)

Thus $\frac{dy}{dx} = \frac{4x^2+3y^2}{2xy} = 2\left(\frac{x}{y}\right) + \frac{3}{2}\left(\frac{y}{x}\right)$ (01), Let $y = ux$; $\frac{dy}{dx} = u + x\frac{du}{dx}$ (01)

Therefore $u + x\frac{du}{dx} = \frac{2}{u} + \frac{3}{2}u$ or $x\frac{du}{dx} = \frac{2}{u} + \frac{u}{2} = \frac{u^2+4}{2u}$ (01)

$\Rightarrow \int \frac{2u}{u^2+4}\,du = \int \frac{dx}{x} \Rightarrow Ln(u^2+4) = Ln|x| + C_1 \Rightarrow u^2+4 = CX$ (01)

$\Rightarrow \frac{y^2}{x^2} + 4 = CX$ (01) and Hence $\boxed{y^2 + 4x^2 = cx^3}$ (01)

Exercise2: (06pts)

Solve the initial value problem:

$$ty' = y - e^{1/t}\ ,\ y(1) = 1.$$

$ty' - y + e^{\frac{1}{t}} = 0 \Rightarrow ty' - y = -e^{\frac{1}{t}}$ (01). S.F: $y' - \frac{1}{t}y = -\frac{1}{t}e^{\frac{1}{t}}$, $P(t) = -\frac{1}{t}$ (01)

I.F: $e^{\int P(t)dt} = e^{\int -\frac{1}{t}dt} = e^{-Ln|t|} = \frac{1}{t}$ $(t>0)$. (01)

Thus: $\frac{1}{t}y' - \frac{1}{t^2}y = -\frac{1}{t^2}e^{\frac{1}{t}} = \frac{d}{dt}\left(\frac{1}{t}y\right) \xrightarrow{\text{Integr.}} \frac{1}{t}y = e^{\frac{1}{t}} + C$ (01) (01)

Hence $\boxed{y = t\left(e^{\frac{1}{t}} + C\right)}$ $y(1) = 1 \Rightarrow 1 = e + C \Rightarrow \boxed{C = 1-e}$ (01)

Exercise3: (07pts)

Determine whether the (DE): $\underbrace{(xy\cos x - 2y\,\sin x)}_{M(x,y)}\,dx + \underbrace{2x\,\sin x}_{N(x,y)}\,dy = 0$ is exact or not.
It it is exact **solve it** , if **not** find an integrating factor which make it exact.

- $\frac{\partial M}{\partial y} = x\cos x - 2\sin x$ (01) $\neq \frac{\partial N}{\partial x} = 2\sin x + 2x\cos x$ (01) $\Rightarrow$ The (DE) is not Exact. (01)
- $\frac{M_y - N_x}{N} = \frac{x\cos x - 2\sin x - 2\sin x - 2x\cos x}{2x\sin x}$ (01) $= -\frac{1}{2}\frac{\cos x}{\sin x} - \frac{2}{x}$ (depend only on x).

Hence The integ. Factor is: $\mu(x) = e^{\int\frac{(M_y-N_x)}{N}dx}$ (01) $= e^{Ln\left(\frac{1}{x^2\sqrt{|\sin x|}}\right)} = \frac{1}{x^2\sqrt{|\sin x|}}$

$\boxed{\mu(x) = \frac{1}{x^2\sqrt{|\sin x|}}}$ (02)