Part I [76 pts] (Written: Provide all necessary steps required in the solution.)

Q1. Find the **extreme values** of $f(x,y) = x\,y^2$ on the **ellipse** $2x^2 + y^2 = 6$ by using the **Lagrange multipliers.** (10 pts)

Q2. **(i)** Find the **Cartesian Equation** of the **parametric curve** $x = 2\cos 2t,\ y = 3\sin 2t;\ -\pi/4 \le t \le \pi/4$.
(4 pts)

(ii) **Sketch** the above curve in the xy-plane. **Indicate** with an arrow the direction in which the curve is traced as the parameter t increases. (4 pts)

(iii) Set up the integral that represents the **surface area** of the solid obtained by revolving the above curve **about the y-axis.** (4 pts)

Q3. If $f(x,y,z)=\ln(zx^2y^2), x=\cos t\theta, y=\ln(t^2+\theta), z=\theta^2,$ find $\partial f/\partial t$ at $(t,\theta)=(1,\pi)$. (8 pts)

Q4. Find the **equation of the plane** that contains the points $(1,2,-3),(3,1,0),$ and $(2,3,-7)$. (8 pts)

Q5. Consider the function $f(x, y) = x^2 + y^2 + x^2 y + 4$.

(i) Show that (0,0), $(\sqrt{2}, -1)$ and $(-\sqrt{2}, -1)$ are the **critical points** of $f(x,y)$. (4 pts)

(ii) Find the **local minimum and maximum value(s)** and the **saddle point(s)** of $f(x,y)$, if any. (6 pts)

Q6. Find the **value of the integral** $\iiint_E xy\,dV$ where E is **bounded by** the parabolic cylinders $x = y^2$ and $y = x^2$, and the planes $z = 0$ and $z = x + y$. (10pts)

Q7. A solid lies within both the cylinder $x^2 + y^2 = 1$ and the paraboloid $z = 4 - x^2 - y^2$, and above the *xy*-plane. Only **Set up** the integral in **cylindrical coordinates** that represents the volume of the solid.

(8 pts)

Q8. Evaluate the integral $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\,dzdydx$. (10 pts)

Part II (8 MCQ: 8 pts/each**)** **Encircle** your **Choice** for each MCQ on the **front page** of your answer book**)**

Q1. If A is the area inside the circle $r = 3\cos\theta$ and outside the cardioid $r = 1 + \cos\theta$, then 2**A=**

(a) $3\pi/2$

(b) 3π

(c) 2π

(d) π

(e) $\pi/2$

Q2. If $|\vec{a}| = 1, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 2,$ then $|\vec{a} \times \vec{b}|^2 =$

(a) 25

(b) 15

(c) 8/3

(d) 25/3

(e) 5

Q3. The $\lim\limits_{(x,y)\to(0,0)} \dfrac{3(x^2+y^2)}{\sqrt{x^2+y^2+1}-1}$

(a) = – 1/2

(b) = 6

(c) = 0

(d) = 4

(e) does not exist

Q4. If $ax + by + cz + d = 0$ is the equation of the tangent plane to the surface $z=\sqrt{9-x^2-y^2}$ at (2, – 2,1), then **(*a* + *b* + *c* + *d*) / 2** equals

(a) – 4

(b) 7/2

(c) –5

(d) – 16

(e) 0

Q5. If the rate of change of $f(x,y) = x e^{y}$ at the point P(2,0) in the direction from P to Q(1/2 , 2) is **V,** then **2V =**

(a) 2

(b) $2\sqrt{2}$

(c) – 3/5

(d) $5\,e^2$

(e) 4

Q6. If D is a triangular region with vertices (0, 0), (0, 2) and (2, 2), then the value of the integral $\iint_D 2xy\,dA$ is

(a) 2

(b) 12

(c) 8

(d) 1

(e) 4

Q7. The value of the integral $\int_0^1 \int_{\sqrt{y}}^1 6e^{x^3}\, dx\, dy$ is

(a) $e\,(e-1)$

(b) $e/2$

(c) $2\,(e-1)$

(d) $e-2$

(e) $3\,(e-1)$

Q8. The iterated integral $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x^2}{1+x^2+y^2}\, dy dx$ when changed to polar coordinates can be expressed as

(a) $\int_0^{\pi} \int_0^1 \frac{r^3 \cos^2\theta}{1+r^2}\, dr d\theta$

(b) $\int_0^{\pi/2} \int_0^{2\cos\theta} \frac{r^4 \cos^3\theta}{1+r^2}\, dr d\theta$

(c) $\int_0^{\pi/2} \int_0^{4\cos\theta} \frac{r^2 \cos^2\theta}{1+r^2}\, r dr d\theta$

(d) $\int_0^{\pi} \int_0^{\cos\theta} \frac{\cos^2\theta}{1+r^2}\, r dr d\theta$

(e) $\int_0^{\pi/2} \int_0^{2\cos\theta} \frac{r^3 \cos^2\theta}{1+r^2}\, dr d\theta$