Part I [52 pts] **(Written: Provide all necessary steps required in the solution.)**

Q1. (i) Find an equation of the plane passing through the points (1,2,3), (– 1 ,2,0) and perpendicular to the plane $x + 2y + 3z = 1$. (7 pts)

(ii) Find the **distance** between the **planes**: $x - 2y + 3z = 1$ and $-2x + 4y - 6z = 1$. (5 pts)

Q2. Let $f(x,y)=\ln\sqrt{16-4x^2-y^2}$

(i) Find and sketch the domain of *f*. (3+2 pts)

(ii) Find the range of *f*. (2 pts)

(iii) Write an equation of the level curve of *f* which passes through the point (1,1). (3 pts)

Q3. Find **parametric equations** of the **normal line** to the **surface** $\ln\left(\dfrac{x}{y-z}\right) = x - 1$ at the point (1,4,3).

(8 pts)

Q4. The values of $z = f(x, y)$ and its partial derivatives at (2, – 2) are given in the following table:

$f(2, -2)$	$f_x(2, -2)$	$f_y(2, -2)$	$f_{xx}(2, -2)$	$f_{yy}(2, -2)$	$f_{xy}(2, -2)$
2	– 5	3	4	7	– 3

If $x = r^2 + s^2$ and $y = 2rs$, then find

(i) $\left.\dfrac{\partial z}{\partial s}\right]_{(r,s)=(1,-1)}$ (4 pts)

(ii) $\left.\dfrac{\partial^2 z}{\partial r \partial s}\right]_{(r,s)=(1,-1)}$ (6 pts)

Q5. Find the **absolute maximum** and **absolute minimum** of $f(x,y) = x(y^2 - 1)$ on the **region** $D = \{(x,y) : x^2 + y^2 \le 28\}$. (12 pts)

Part II (8 MCQ: 6pts/each**)** **Encircle** your **Choice** for each MCQ on the **front page** of your answer book**)**

Q1. Let (a,b,c) is the **point of intersection** of the lines

$$\boldsymbol{L_1}\text{:}\quad x = 4t + 2,\quad y = 5t,\quad z = 1 - t \quad\text{and}\quad \boldsymbol{L_2}\text{:}\quad \frac{x-2}{2} = \frac{y-3}{2} = z + 8.$$

Then $\boldsymbol{a + b + c}$ is

(a) – 23

(b) 27

(c) 18

(d) 31

(e) – 33

Q2. The **surface** given by $x^2 + y^2 - z^2 - 4x + 2y - 2z + 5 = 0$ is

(a) a paraboloid.

(b) a hyperboloid of one sheet.

(c) an ellipsoid.

(d) a hyperboloid of two sheets.

(e) an elliptic cone.

Q3. In order to make $f(x,y) = \dfrac{x^2 + y^2 - 4x^4y^4}{2(x^2 + y^2)}$ **continuous** at the **origin**, we must define $f(0,0)$ as

(a) – 3/2

(b) – 1

(c) 0

(d) 1/2

(e) 3/2

Q4. If $f(x,y)=(x^2+y)\cos(x^2+y)$ then $f_{xy}\left(\sqrt{\frac{\pi}{2}},\pi\right)$ **equals to**

(a) 0

(b) $4\sqrt{\pi}$

(c) $2\sqrt{2\pi}$

(d) $\pi\sqrt{\pi}\,/\,2$

(e) $6\pi\sqrt{\pi}$

Q5. If $f(x,y,z)$ be a **differentiable** function with

$$f(1,-2,3)=1,\ f_x(1,-2,3)=10,\ f_y(1,-2,3)=100, \text{ and } f_z(1,-2,3)=20.$$

Then using the **linear approximation** of f at (1,– 2,3), the **approximate value** of f(0.9, – 2.02,2.95) is

(a) 7

(b) 310

(c) – 8

(d) – 5/4

(e) – 3

Q6. The **equation of the curve** $\ln(u^2+v^4+w^2)-\sqrt{w^2+3}=\ln 3-2$ defines w as an **implicit** function of u and v. Then $\partial w/\partial v$ at (u,v,w) = (1,1,1) is

(a) – 8

(b) ln2 – 2

(c) 2ln2 – 5

(d) 1/3

(e) 8/5

Q7. The **temperature of a solid** at a point (x,y,z) is given by $T(x,y,z)=(x+2y+3z)^{3/2}$. Then the rate **of change** of temperature at the point (1,1,2) in the direction of vector $\vec{v}=2\vec{j}-\vec{k}$ is

(a) $9/3\sqrt{7}$

(b) $7/\sqrt{5}$

(c) 4/3

(d) $-2/\sqrt{7}$

(e) $9/(2\sqrt{5})$

Q8. A function $g(x,y)$ has continuous first and second order partial derivatives. Let (1,1) and (3,0) are the critical points of g with the values given in the following table

(x,y)	**$g_x(x,y)$**	**$g_y(x,y)$**	**$g_{xx}(x,y)$**	**$g_{yy}(x,y)$**	**$g_{xy}(x,y)$**
(1,1)	0	0	– 2	– 2	– 1
(3,0)	0	0	0	– 6	3

Then g has

(a) a local minimum at (1,1) and a saddle point at (3,0).

(b) a local maximum at (1,1) and a saddle point at (3,0).

(c) a local minimum at (1,1) and a local maximum at (3,0).

(d) a local maximum at (1,1) and a local minimum at (3,0).

(e) a saddle point at (1,1) and a local minimum at (3,0).