Part I [52 pts] **(Written: Provide all necessary steps required in the solution.)**

Q1. (i) Find d^2y/dx^2 for the parametric curve **C:** $x = 3t^2 - t, \quad y = 2t + t^3$. (5+5 pts)

(ii) Find the interval (s) where **C** is **concave up**.

Q2. Consider the vectors $\vec{u} = -3\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{v} = \vec{i} + 2\vec{j} - 3\vec{k}$ (10 pts)

(i) Find the angle between $\vec{u}$ and $\vec{v}$.

(ii) Find the projection of $\vec{u}$ onto $\vec{v}$

Q3. Use the **scalar triple product** to determine whether the four points: (10 pts)

$$A(1,3,2),\ \ B(3,-1,6),\ \ C(5,2,0),\ \ D(3,6,-4)$$

lie in the **same plane**.

Q4. Find the exact **length** of the **polar curve**: $r=\theta^2, \quad 0\le\theta\le\pi/4$. (10 pts)

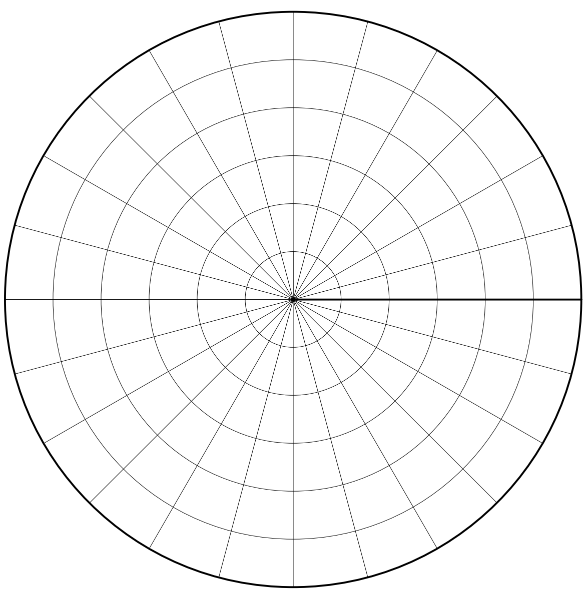
Q5. Consider the polar curve **C**: $r = 2 + 4\sin\theta$ (3 + 2 + 2 +5 pts)

(a) Show that **C** is **symmetric** about the vertical line $\theta = \dfrac{\pi}{2}$.

(b) Find the **polar coordinates** of the points where C **intersects** the **polar axis**.

(c) Find the **polar coordinates** of the points where C **intersects** the lines $\theta = \dfrac{\pi}{2}$ and $\theta = \dfrac{\pi}{4}$

(d) **Plot** the points obtained in (b)-(c) and make use of (a) to **sketch the graph** of **C** in the following polar chart**:** [*Indicate **important values** of* r and θ *in the **outer circle of the chart***]



Part II (8 MCQ: 6pts/each**)** **Encircle** your **Choice** for each MCQ on the **front page** of your answer book**)**

Q1. If the end points of a diameter of a sphere lie at $A(1,4,-2)$ and $B(-7,1,2)$ then an equation of the sphere is given by

(a^*) $x^2+y^2+z^2+6x-5y=7$

(b) $x^2+y^2+z^2-8x-4y=10$

(c) $x^2+y^2-z^2+6x-4y=7$

(d) $x^2+y^2+z^2+7x-10y=20$

(e) $x^2+y^2+z^2+6x+4y=12$

Q2. Suppose that a 3-D vector $\vec{v}$ lies below the *xy*-plane and has the **direction angles** α, β, γ with *x*, *y* and *z* axes respectively. If $\alpha=\dfrac{\pi}{4},\ \beta=\dfrac{\pi}{3}$, then the value of γ is given by

(a^*) $2\pi/3$

(b) $(\sqrt{2}\,\pi)/2$

(c) $-1/2$

(d) $5\pi/6$

(e) $-1/\sqrt{2}$

Q3. A **value of** α for which the vectors $\vec{u} = 3\vec{i} + \alpha\vec{k}$ and $\vec{v} = 2\alpha\vec{i} - \vec{j}$ have the **same length** is given by

(*a**) $\sqrt{8/3}$

(*b*) $\sqrt{5/3}$

(*c*) $\sqrt{8/5}$

(*d*) $\sqrt{7/3}$

(*e*) $\sqrt{5/8}$

Q4. The area of the triangle with the vertices $(a,0,0)$, $(0,2a,0)$ and $(0,0,3a)$ is

(*a**) $7a^2/2$

(*b*) $5a^2/2$

(*c*) $6a^3$

(*d*) $7a$

(*e*) $3a^3/2$

Q5. The area of the region inside the curve $r = 3\sin\theta$ and outside the curve $r = 2 - \sin\theta$ is

(a^*) $\int_{\pi/6}^{5\pi/6} (4\sin^2\theta + 2\sin\theta - 2)\,d\theta$

(b) $\int_{-\pi/6}^{\pi/6} (2\sin^2\theta - 2\sin\theta - 1)\,d\theta$

(c) $\int_{\pi/6}^{5\pi/6} (4\sin^2\theta + \sin\theta + 3)\,d\theta$

(d) $\int_{\pi/6}^{5\pi/6} (4\sin^2\theta + 5\sin\theta - 2)\,d\theta$

(e) $\int_{-\pi/6}^{\pi/6} (4\sin^2\theta + 5\sin\theta - 2)\,d\theta$

Q6. The Cartesian equation of the curve $x = \ln t,\ y = \sqrt{t},\ t \geq 1$ is given by

(a^*) $y = e^{x/2},\quad x \geq 0$

(b) $y = e^{x},\quad x \geq 1$

(c) $y = e^{x/2},\quad x \geq 1$

(d) $y = e^{x},\quad x \geq 0$

(e) $y = e^{2x},\quad x \geq 0$

Q7. The slope of the tangent line to the polar curve $r = \cos\theta + 1$ at $\theta = \pi/2$ is

(a^*) 1
(b) $1/2$
(c) $1/3$
(d) 0
(e) $-1/2$

Q8. Two **forces** ***F*** and **G** are acting on an object placed at the **origin** of the ***xy*-plane** with **magnitudes** 1 N and 2 N respectively.

If ***F*** acts along the **positive** *y-axis* and **G** makes an **angle** of $\theta = \pi/3$ with the **positive** *x*-axis, then the **magnitude** of the **resultant** force ***F* + *G*** is

(a^*) $\sqrt{5+2\sqrt{3}}$ N
(b) $\sqrt{1+\sqrt{3}}$ N
(c) $\sqrt{2+\sqrt{3}}$ N
(d) $5+\sqrt{3}/2$ N
(e) $\sqrt{2+2\sqrt{3}}$ N