Name: _____   ID number: _____

1.)(5pts)The point $(4, -4, 2)$ is given in rectangular coordinate. Find its cylindrical coordinates.

3.)(5pts)Find the volume of the solid which lies below the sphere $x^2 + y^2 + z^2 = 4$, above the $xy-$plane and inside the cylinder $x^2 + y^2 = 3$.
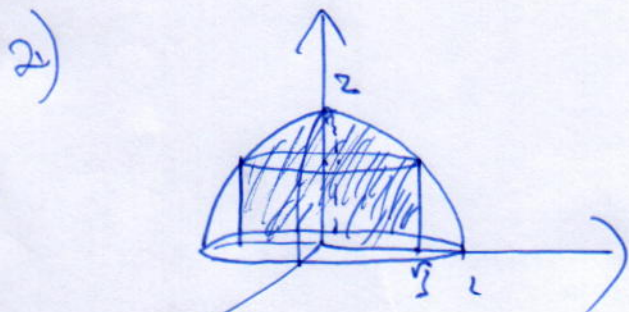
1) (r, θ, z)

$r = \sqrt{x^2 + y^2} = \sqrt{16 + 16} = 4\sqrt{2}$

$\cos\theta = \frac{x}{r} = \frac{4}{4\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin\theta = \frac{y}{r} = \frac{-4}{4\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\Rightarrow \theta = -\pi/4 + 2k\pi$

$\left(4\sqrt{2}, -\pi/4, 2\right)$

2)

$V = \int_0^{2\pi} \int_0^{\sqrt{3}} \int_0^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta$

$= \int_0^{2\pi} \int_0^{\sqrt{3}} r \sqrt{4-r^2} \, dr \, d\theta$

$= \int_0^{2\pi} \left[-\frac{1}{3} (4-r^2)^{3/2} \right]_0^{\sqrt{3}} d\theta$

$= \frac{7}{3} 2\pi$

$= \frac{14\pi}{3}$.

Name: _____

ID number: _____

1.)(5pts)The point $(4, 4\sqrt{3}, -7)$ is given in rectangular coordinate. Find its cylindrical coordinates.

2.)(5pts)Find the volume of the solid which lies inside the sphere $x^2 + y^2 + z^2 = 4$ and between the cones $\phi = \frac{\pi}{3}$ and $\phi = \frac{2\pi}{3}$.

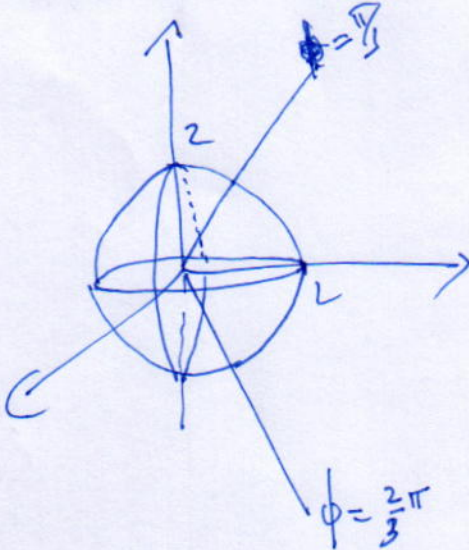1.) (r, θ, z)

$r = \sqrt{x^2 + y^2} = \sqrt{16 + 3 \times 16} = 8$

$\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$

$\begin{cases} \cos\theta = \frac{4}{8} = \frac{1}{2} \\ \sin\theta = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2} \end{cases}$

$\theta = \pi/3$

$\Rightarrow \boxed{\left(8, \frac{\pi}{3}, -7\right)}$

2.)

$V = \int_0^{2\pi} \int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \int_0^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$= \left[\frac{\rho^3}{3}\right]_0^2 \left[-\cos\phi\right]_{\pi/3}^{\frac{2\pi}{3}} \quad 2\pi$

$= \frac{8}{3}\left(\frac{1}{2} + \frac{1}{2}\right) 2\pi$

$= \frac{16}{3}\pi$