MATH 201.1 (Term 111)

Quiz 3 (Sects. 12.5-6) Duration: 15mn

Name: ID number:

1.)(4pts)Find an equation of the plane that passes through the point $A(1,0,-1)$ and contains the line $x = 1 - t$, $y = 2 + 4t$, $z = t$.

2.)(6pts)Identify and sketch the surface $3x^2 - y^2 + 4z^2 - 4z + 5 = 0$.

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1) Let $B(1,2,0)$ obtained by taking $t=0$ in the equation of the line. The vectors $\overrightarrow{AB}$ and $\vec{u} = \langle -1, 4, 1 \rangle$ belong to the plane, and $\overrightarrow{AB} \times \vec{u}$ is a normal vector to the plane.

$$\overrightarrow{AB} = \langle 0, 2, 1 \rangle$$

$$\overrightarrow{AB} \times \vec{u} = \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ -1 & 4 & 1 \end{vmatrix}$$

$$= \langle -2, -1, 2 \rangle$$

The equation of the plane is

$$-2(x-1) - (y) + 2(z+1) = 0$$

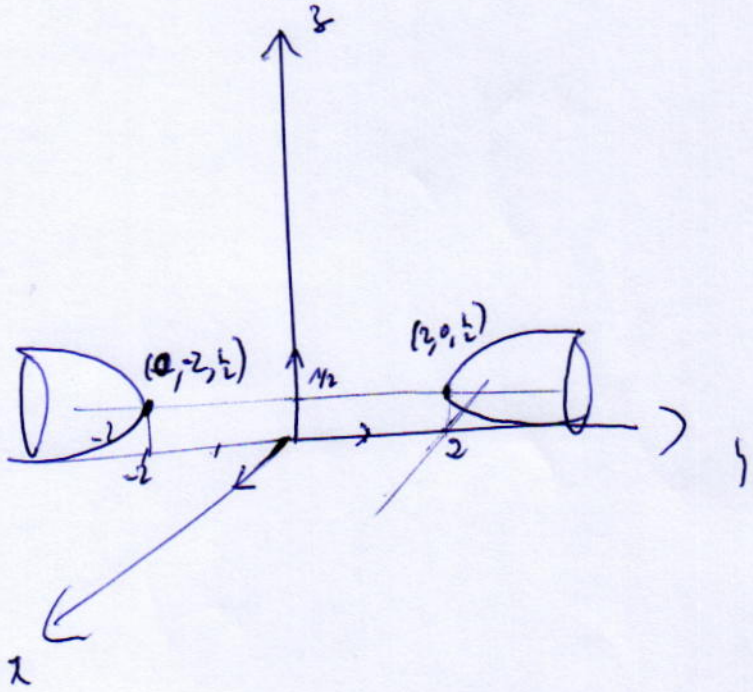
$$\boxed{-2x - y + 2z + 4 = 0}$$

2) $3x^2 - y^2 + 4z^2 - 4z + 5 = 0$

$$3x^2 - y^2 + 4\left(z - \frac{1}{2}\right)^2 - 1 + 5 = 0$$

$$3x^2 + 4\left(z - \frac{1}{2}\right)^2 = y^2 - 4$$

This is an Hyperboloid of two sheets.



MATH 201.5 (Term 111)

Quiz 3 (Sects. 12.5-6) Duration: 15mn

Name: ID number:

1.)(4pts)Find an equation of the plane that passes through the points $A(1,1,1)$ and $B(-1,0,2)$ and that is perpendicular to the plane $x-2y+z=2$.

2.)(6pts)Identify and sketch the surface $2x^2-y^2+z^2-4y-2z-3=0$.

1.)

The vectors $\overrightarrow{AB}$ and $\overrightarrow{n_1}=\langle 1,-2,1\rangle$ belong to the plane, so that $\overrightarrow{AB}\times\overrightarrow{n}$ is normal to the plane.

$$\overrightarrow{AB}\times\overrightarrow{n}=\begin{vmatrix} i & j & k \\ -2 & -1 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$=\langle 1,3,5\rangle$$

The equation of the plane is

$$(x-1)+3(y-1)+5(z-1)=0$$

$$\boxed{x+3y+5z-9=0}$$

2) $2x^2-y^2+z^2-4y-2z-3=0$

$$2x^2-(y+2)^2+4+(z-1)^2-1-3=0$$

$$2x^2+(z-1)^2=(y+2)^2$$

This is a cone.