MATH 201.1 (Term 111)

Quiz 2 (Sects. 12.1-4) Duration: 20mn

Name: **ID number:**

1.)(4pts)Find the center and radius of the sphere of equation $x^2+y^2+z^2+\frac{1}{4}x-z=0$. What are the intersection points of this sphere with the $xy-$plane.
2.)(3pts)Find the area of triangle of vertices $A(0,1,1)$, $B(2,-1,1)$ and $C(-1,1,0)$. What is the angle between the vectors $\vec{AB}$ and $\vec{AC}$.
3.)(3pts)Find the volume of the box defined by the vectors $a=<0,-1,1>$, $b=<2,1,-1>$ and $c=<1,-1,0>$.

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1) $x^2+\frac{x}{4}+y^2+z^2-z=0$

$(x+\frac{1}{8})^2-\frac{1}{64}+y^2+(z-\frac{1}{2})^2-\frac{1}{4}=0$

$(x+\frac{1}{8})^2+y^2+(z-1/2)^2=\frac{17}{64}$

this is a sphere of centre $\begin{pmatrix}-1/8\\0\\1/2\end{pmatrix}$ and radius $\frac{\sqrt{17}}{8}$.

The xy-plane has equation $z=0$. We substitute $z=0$ into the equation of the sphere. We find

$(x+\frac{1}{8})^2+y^2+\frac{1}{4}=\frac{1}{64}+\frac{1}{4}$

$\Rightarrow (x+\frac{1}{8})^2+y^2=(\frac{1}{8})^2$

This is a circle of centre $\begin{pmatrix}-1/8\\0\\0\end{pmatrix}$ and radius $1/8$.

2) $A=\frac{|\vec{AB}\times\vec{AC}|}{2}$

$\vec{AB}=\langle 2,-2,0\rangle$

$\vec{AC}=\langle -1,0,-1\rangle$

$\vec{AB}\times\vec{AC}=\begin{vmatrix} i & j & k\\ 2 & -2 & 0\\ -1 & 0 & -1\end{vmatrix}$

$=\langle 2,2,-2\rangle$

$\boxed{A=\frac{\sqrt{4+4+4}}{2}=\sqrt{3}}$.

$\vec{AB}\cdot\vec{AC}=|\vec{AB}|\,|\vec{AC}|\cos\alpha$

$\cos\alpha=\frac{-2}{\sqrt{2}\sqrt{8}}=-1/2$

$\theta=2\pi/3$

3) $V=|a\cdot(b\times c)|$

$b\times c=\begin{vmatrix} i & j & k\\ 2 & +1 & -1\\ 1 & -1 & 0\end{vmatrix}$

$=\langle -1,-1,-3\rangle$

$a\cdot(b\times c)=1-3=-2$

$\boxed{V=2}$

MATH 201.5 (Term 111)

Quiz 2 (Sects. 12.1-4) Duration: 20mn

Name: ID number:

1.)(4pts)Find the center and radius of the sphere of equation $x^2+y^2+z^2-x+\frac{1}{2}y=0$. What are the intersection points of this sphere with the xz-plane.

2.)(3pts)Find the vector projection of $u=<1,-1,1>$ onto $v=<0,3,1>$. What is the angle between the vectors u and v.

3.)(3pts)Find the volume of the box defined by the vectors $a=<1,0,1>$, $b=<0,1,-2>$ and $c=<1,1,1>$.

1.) $x^2 - x + y^2 + \frac{1}{2}y + z^2 = 0$

$(x-\frac{1}{2})^2 - \frac{1}{4} + (y+\frac{1}{4})^2 - \frac{1}{16} + z^2 = 0$

$(x-\frac{1}{2})^2 + (y+\frac{1}{4})^2 + z^2 = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}$

This is a sphere of centre $\begin{pmatrix}1/2\\-1/4\\0\end{pmatrix}$ and radius $\frac{\sqrt{5}}{4}$.

The xz-plane has equation $y=0$.
We substitute into the equation of the sphere. We find

$(x-\frac{1}{2})^2 + \frac{1}{16} + z^2 = \frac{1}{4} + \frac{1}{16}$

$\Rightarrow (x-\frac{1}{2})^2 + z^2 = \frac{1}{4} = (\frac{1}{2})^2$

This is a circle of centre $\begin{pmatrix}1/2\\0\\0\end{pmatrix}$ and radius $1/2$.

2.) $\text{proj}_v^u = \frac{u \cdot v}{|v|^2} v$

$= \frac{-3+1}{10}\langle 0,3,1\rangle$

$= \langle 0, \frac{-3}{5}, -\frac{1}{5}\rangle$

b) $\cos\alpha = \frac{u \cdot v}{|u||v|}$

$= \frac{-2}{\sqrt{3}\sqrt{10}} = -\frac{\sqrt{30}}{15}$

$\alpha = \cos^{-1}(-\frac{\sqrt{30}}{15})$

3.) $V = |a\cdot(b\times c)|$

$b\times c = \begin{vmatrix} i & j & k \\ 0 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix}$

$= \langle 3, -2, -1\rangle$

$a\cdot(b\times c) = 3-1 = 2$

$\boxed{V=2}$